Limited-Angle Tomography Using Constrained Sinogram Restoration[1]

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Summary

Limited-angle tomography involves reconstructing images from projections that are available only over a limited angular range. This problem arises in a number of disciplines including medical imaging, non-destructive evaluation, and geophysical and sonar imaging. Because of the lack of sufficient data, the success of any reconstruction method relies upon the availability and utilization of prior knowledge. In this paper, we present an algorithm that calculates the maximum *a posteriori* estimate of the complete sinogram, using prior knowledge of the smoothness of the sinogram, fundamental mathematical constraints on the Radon transform, and a complete probabilistic characterization of the observation noise. The object is reconstructed using convolution backprojection applied to the restored sinogram.

Many objects of interest tend to have smooth sinograms, although the objects themselves may not be smooth. We incorporate this observation by defining a Markov random field prior probability on full sinograms, rather than on objects. The Markov random field we use is of the simplest kind — nearest-neighbor, with quadratic potential terms — although more elaborate models may be used. The fundamental mathematical constraints on the Radon transform were described by Ludwig in 1966. In summary, a valid Radon transform must obey a certain periodicity property and an infinite set of integral constraints. Both of these properties may be incorporated by constraining the feasible configurations of the Markov random field to be those sinograms that satisfy these constraints.

Using a known noise model (zero-mean, Gaussian), the maximum *a posteriori* solution to the sinogram restoration problem may be formulated. The solution to this problem is a constrained optimization algorithm, and because of the simple form of both the prior and the observation noise, we were able to develop an iterative primal-dual algorithm that converges quite rapidly to the desired solution. Experimental results are included to show the importance of the various parts of the formulation and to show the overall performance of several simulation results.

[1]This research was supported by the National Science Foundation grants ECS-83-12921 and ECS-87-00903 and the U.S. Army Research Office grants DAAG29-84-K-0005 and DAAL03-86-K-0171. In addition, the work of the first author was partially supported by a U.S. Army Research Office Fellowship.

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