SEGMENTATION AND MAPPING OF HIGHLY CONVOLUTED CONTOURS WITH APPLICATIONS TO MEDICAL IMAGES

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ABSTRACT

In this paper we present a method that simultaneously identifies the central layer of the human cortex and maps it onto the interval [0, 1] of the real axis. Statistical and geometric information is incorporated into a global variational problem, whose solution is obtained iteratively. The method is evaluated on a set of magnetic resonance (MR) images, acquired with a protocol that optimizes the contrast between the cortical grey matter and the underlying white matter.

INTRODUCTION

Since the advent of outstanding anatomical and functional imaging methods, such as magnetic resonance imaging and positron emission tomography (PET), respectively, interest in the development of a map of the human cortical surface has sharply increased for several reasons. First, a large amount of anatomical information that has been published for several decades in the neurophysiology literature can be placed in a common reference frame: the mapped surface. Second, superposition of functional information that is extracted from PET or magnetoencephalography data can be superimposed on the map, revealing interrelations of the brain functions. Third, geodesic and area measurements on the cortical surface are made possible. Finally, this map is expected to become part of a diagnostic tool for brain abnormalities.

Various segmentation techniques have been developed in the past; however, none of them is suitable to our problem. Popular techniques like statistical segmentation, relaxation labeling, Markov random fields or region growing, can successfully yield a set of points which belong to the cortex; but they don't provide a map. Previous attempts to map the cortical surface suffer from either the need for strong human intervention [1], or the requirement of a triangularization of the surface [2], which assumes that we already have a reconstructed surface and a (non-regular) grid on it.

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In contrast to these methods, we pose a global optimization problem and solve it numerically, minimizing human intervention. We show that under certain conditions this problem has a unique solution and that our iterative solution converges to it. Our current results apply only to the mapping of the contours of the cross-sections of the cortex. However, the algorithm can be easily extended to the three dimensional case, in which the cortical surface is mapped to the unit square.

VARIATIONAL FORMULATION

Our approach is a variational approach which takes into consideration the geometry of the cortex. In particular, we model the cortex as a thick sheet that is "sandwiched" between cerebrospinal fluid and white matter (see Fig. 1). We also assume that its thickness

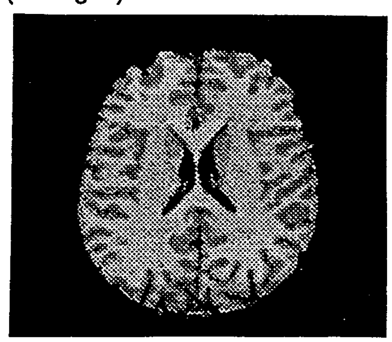


Figure 1: An MR image of a cross-section of the human brain.

is relatively uniform and its statistics are known and are distinct from those of the other tissues present in the brain. Given these statistics, we define the *cortical mass function* to be the posterior probability that an image point belongs to the grey matter given the image intensity $I(x,y)$:

$$\begin{aligned} m(x,y) &= Pr[(x,y) \in GM \mid I(x,y)] & (1) \\ &= \frac{f(I(x,y) \mid (x,y) \in GM) Pr[GM]}{f(I(x,y))} & (2) \\ f(I(x,y)) &= \sum_{i=1}^{3} f(I(x,y) \mid \lambda_i) Pr(\lambda_i) & (3) \end{aligned}$$

with $\{\lambda_1, \lambda_2, \lambda_3\} = \{GM, WM, BG\}$, and GM, WM, BG denoting the grey matter, white matter and background respectively. The conditional distributions $f(I(x,y)|\lambda_i)$ are assumed to be Gaussian. We define the exact solution to the contour reconstruction problem to be the central layer of this cortical mass. Incorporating this geometric model into a variational formulation, we introduce an energy functional that forces the contour points to balance whenever they sit on the center of cortical mass within a circular neighborhood around them. We also introduce a regularization term, inherent to inverse problems of this kind, which reduces the effects of noise and large voxel size by favoring curves which are smooth and can elastically deform. This regularization term favors a solution in which the points are equidistantly distributed along the curve, which in turn yields an isometric mapping of the curve to the unit interval.

We formally state the problem as follows. Let $\mathcal{F} \subset \mathbb{R}^2$ be the image of the curve to be reconstructed and $\mathcal{N}$ be a circular neighborhood of radius R around the point $(x,y) \in \mathcal{F}$. We identify $(c_x(x,y), c_y(x,y))$ as the center of the cortical mass included in $\mathcal{N}$, as shown in Fig. 2. We seek a map $\mathcal{M} : [0,1] \rightarrow \mathbb{R}^2$ which maps

Figure 2: The mass and center of mass functions

the point $s \in [0,1]$ to $\mathcal{M}(s) = (x(s), y(s)) \in \mathcal{F}$ and define our estimate of this mapping by the following minimization problem:

$$(\hat{x}(s), \hat{y}(s)) = \underset{x,y}{\operatorname{argmin}} \, \mathcal{E} = \underset{x,y}{\operatorname{argmin}} \, \mathcal{E}_P + \mathcal{E}_B \quad (4)$$

$$\mathcal{E}_P = \int_s \left[(x - c_x(x,y))^2 + (y - c_y(x,y))^2 \right] ds \quad (5)$$

$$\mathcal{E}_B = K_o \int_s \left[\left(\frac{dx}{ds} \right)^2 + \left(\frac{dy}{ds} \right)^2 \right] ds \quad (6)$$

subject to the boundary conditions:

$$x(0) = \alpha \;\;,\;\; x(1) = \beta \;\;,\;\; y(0) = \gamma \;\;,\;\; y(1) = \delta \quad (7)$$

For simplicity we have set $x = x(s)$, $y = y(s)$. The $\mathcal{E}_P$ term forces the points of the curve to follow the center of the mass that is within their neighborhood, while the $\mathcal{E}_B$ term forces them to be uniformly distributed along the curve. From now on we will refer to the above variational problem as (CVP).

The solution to (CVP) must satisfy the (necessary) Euler equations, given by

$$(x - c_x)\left(1 - \frac{\partial c_x}{\partial x}\right) - (y - c_y)\frac{\partial c_y}{\partial x} - K_o \frac{d^2 x}{ds^2} = 0 \quad (8)$$

$$(y - c_y)\left(1 - \frac{\partial c_y}{\partial y}\right) - (x - c_x)\frac{\partial c_x}{\partial y} - K_o \frac{d^2 y}{ds^2} = 0 \quad (9)$$

In addition, the boundary conditions specify the end points of the curve.

The above formulation yields a curve that behaves like a *snake* [3]. It deforms under the presence of external and internal forces; the former leading the curve towards the central layer of the cortical surface contours, the latter maintaining its elastic behavior. This elastic behavior is necessary in order for the curve not to cross itself and to be as isometric as possible. These forces are the physical equivalent of the Euler equations (8) and (9), which imply that the total sum of the forces applied on each point has to be equal to zero at a minimum energy curve. The deformations of the curve are the physical equivalent of the iterative method, which we apply in order to solve equations (8) and (9), as discussed in the next section.

NUMERICAL SOLUTION

To solve (CVP) numerically, we discretize the curve, sampling it with $N+1$ points. Let (x_0, y_0), (x_1, y_1), ..., (x_N, y_N) be the samples of the curve and

$$\mathrm{d} = (x_1, x_2, \ldots, x_{N-1}, y_1, y_2, \ldots, y_{N-1}).$$

The discrete form of the Euler equations is then:

$$K_o N^2 \mathrm{A}\mathrm{d} + \phi(\mathrm{d}) = K_o N^2 \mathrm{b} \quad (10)$$

where

$$\Phi(\mathbf{d}) = \sum_{i=1}^{N-1} \left[(x_i - c_x(x_i, y_i))^2 + (y_i - c_y(x_i, y_i))^2 \right],$$

$$\phi(\mathbf{d}) = \nabla \Phi(\mathbf{d}), \tag{11}$$

$$\mathbf{b} = [\alpha, 0, ..., 0, \beta, \gamma, 0, ..., 0, \delta]^T,$$

and A is a block tridiagonal matrix.

We solve (10) using the Gauss-Seidel iterative method, after precalculating the mass and center of mass functions for the whole image, which drastically decreases the computational time.

CONVERGENCE

Any solution to the discrete Euler equations (10) may be either a local or global minimum, depending on both the geometry of the cortex and the regularization constant K_o. However, if the energy function, which in discrete form is

$$\tilde{\mathcal{E}}(\mathbf{d}) = K_o N^2 (\mathbf{d}^T \mathbf{A} \mathbf{d} - 2\mathbf{b}^T \mathbf{d}) + 2\Phi(\mathbf{d}) \tag{12}$$

is locally convex and the curve is placed sufficiently close to the global minimum, then the iterative process will converge to it. The following theorem gives a condition under which the problem is guaranteed to have a unique solution.

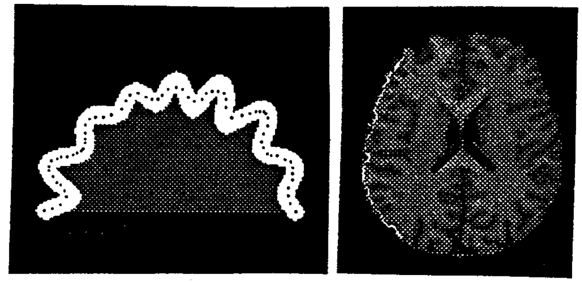
Theorem: *Let $D_{\max}$ be the maximum distance of the curve from the cortical mass. If $D_{\max} < R$ (the neighborhood size), and the regularization constant K_o is chosen such that*

$$K_o > -\frac{2 \min \{ n_i n_i' \kappa_i, i = 1, ..., N-1 \}}{N^2 \lambda_{\min}} \tag{13}$$

then the energy function is convex and, hence, any solution to the Euler equations is a global minimum. In (13), $\lambda_{\min}$ is the minimum eigenvalue of the matrix A, n_i is the distance of point (x_i, y_i) from the center of mass included in its neighborhood, and κ_i is the curvature of the cortical surface contour near (x_i, y_i).

EXPERIMENTAL RESULTS

The algorithm was applied to synthetic and real data and the results are shown in Figs 3a and 3b respectively. From Fig. 3a we can see that the curve (which is superimposed on the data), captures the basic structure of the cross section of the synthetic cortex and balances very close to the central layer of it. An oversmoothing of the very sharp foldings is present, but is not severe. Fig. 3b, however, demonstrates the inability of the curve to follow the very sharp foldings of the human cortex. Although the reconstruction of the outside of the cortex is a successful first step, it is clear that it is not enough. In the next section we will see how modifying the algorithm, can drastically improve the results.



(a) (b)

Figure 3: (a) Reconstructed contour of synthetic data. (b) Reconstructed contour of real data.

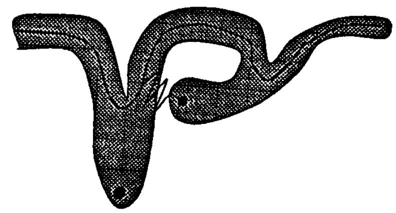


Figure 4: A typical case in which the fixed control point model fails, because the control points are not close to the curve.

CONTROL POINTS

In order to remedy the oversmoothing problem, we introduce the notion of the *control points*. These control points may be placed to the bottom parts of the sharp foldings of the brain (called *sulci*), in order to attract the curve towards them. However, this model of the control points, which is similar to the springs introduced in [3], is inadequate in the case of highly convoluted contours. In Fig. 4 for example, the curve is attracted by the wrong control point. In our formulation, we allow the control points to move in conjunction with the curve, attracting it towards the deep parts of the sulci. A global attractive force, originating in the center of the brain, is exerted on each control point, reflecting the natural tendency of the cortex to fold towards the inside of the brain. Since the control points are not allowed to enter the white matter (see Fig. 1), they slide along the grey-white matter interface and balance at the bottom parts of the sulci.

To incorporate this model of *moving control points* we define m_w as the mass of the *white matter* that is included in the neighborhood of the point (x, y). This mass is a posterior distribution, defined similarly to $m(x, y)$ in (3). Let (c_{xo}, c_{yo}) be the center of the brain (if the sample is centered during the data acquisition, we can take the center of the image) and (c_x^w, c_y^w) be the center of m_w. The position of the control points is updated in every iteration as follows:

$$x^{t+1} = x^t + \delta\,(c_{xo} - x) + a(m_w)\delta\,(c_x^w - x) \qquad (14)$$

$$y^{t+1} = y^t + \delta\,(c_{yo} - y) + a(m_w)\delta\,(c_y^w - y) \qquad (15)$$

where t denotes the iteration number, $a(m_w)$ is an increasing function and δ is a fixed step. This updating formula forces the control points to move towards the point (c_{xo}, c_{yo}), but when they reach the grey-white matter interface they slide along it, instead of penetrating it.

To incorporate the control points into the global variational formulation, we modify the energy function in (4) as follows:

$$(\hat{x}(s), \hat{y}(s)) = \underset{\mathbf{d}}{\operatorname{argmin}}\,\mathcal{E} = \underset{\mathbf{d}}{\operatorname{argmin}}\,\mathcal{E}_F + \mathcal{E}_E + \mathcal{E}_{CP} \quad (16)$$

$$\mathcal{E}_{CP} = \sum_{k=1}^{K} l_k^2 \qquad (17)$$

where l_k is the minimum distance of the kth control point from the curve and K is the number of control points.

Experimental results with control points are demonstrated in Fig. 5. The reconstruction of the contours is clearly better. The curve, attracted by the control points, folds towards the inside of the brain. However, it is also clear that the final solution doesn't reflect the actual structure of the contours in the neighborhood of some of the sulci.

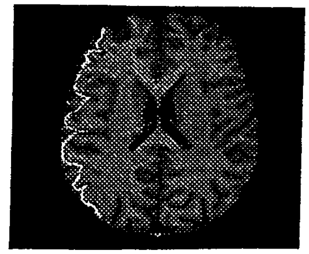


Figure 5: Reconstructed contour using control points.

CONCLUSION

We have developed an algorithm that reconstructs the cross-sections of the central layer of the cortex, and maps it onto the unit interval. We formulate a variational problem which favors solutions that pass through the central layer of cortex, and have samples that are uniformly distributed. We have incorporated two kinds of information into this formulation. The first is geometric information about the structure of the brain. The second is statistical information, derived from the image.

The algorithm performs satisfactorily in the reconstruction of the outside of the contour, but it fails to follow its very sharp foldings. Hence, this formulation is inadequate for the complexity of the problem, although in the case of smoother contours similar formulations have proved to perform successfully (e.g. [4]). We have extended it to include an energy term that favors solutions that include a set of points, called moving control points. These points travel towards the inside of the brain, attracting the curve at the same time, and finally balance at the deep parts of the sulci.

The performance of the algorithm improves dramatically with moving control points, but still does not completely reflect the complexity of the cortical surface. Future research will focus on the identification of points that where a folding occurs and the convexity of the augmented energy function.

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