A NEW EXTERNAL FORCE MODEL FOR SNAKES

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ABSTRACT

Active contour models, or snakes, are widely used in image analysis for boundary identification and mapping. One difficulty with snakes is their inability to converge to concave portions of object boundaries. In this paper we address this problem by proposing a new external force model. This model uses an external force that is a convex combination of the usual external force and a new force derived from an estimate of the local curvature of the object boundary. This force simultaneously pulls the snake toward the boundary and into the concave region. Experimental results are presented.

1. INTRODUCTION AND BACKGROUND

Active contour models, or snakes [1], have been widely used to find object boundaries in images [1–4]. A snake is a curve $\mathbf{x}(s) = [x(s), y(s)]$ which moves through an image to minimize the energy

$$E = \frac{1}{2}\int_0^1 \alpha|\mathbf{x}'(s)|^2 + \beta|\mathbf{x}''(s)|^2 + E_{\text{ext}}(\mathbf{x}(s))ds \qquad (1)$$

where α and β are constants that control the snake's tension and rigidity. The external energy function, E_{ext}, is derived from the image. It is designed to achieve a minimum on the desired contour of interest, and is of primary concern in this paper.

A snake that minimizes E must satisfy the Euler equation

$$\alpha\mathbf{x}''(s) - \beta\mathbf{x}''''(s) - \nabla E_{\text{ext}} = 0 \qquad (2)$$

which can be viewed as a force balance equation. The first two terms provide internal forces which discourage stretching and bending while the third term $-\nabla E_{\text{ext}}$ is the external force which pulls the snake towards the desired image contour. To make the snake dynamic a time derivative is added to the right hand side of (2), and the discretized system is solved iteratively [1].

Snakes have difficulty with concave boundaries such as at the top of the U-shaped figure in Fig. 1 (see also [2,3]).

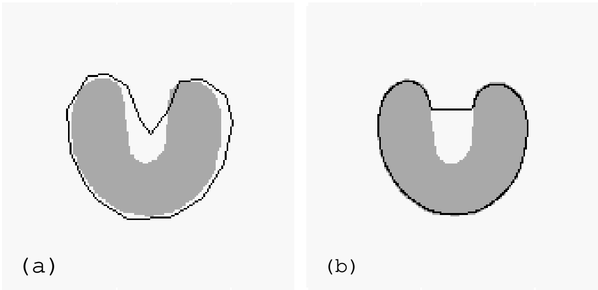

Figure 1: (a) Initial snake and (b) final snake using traditional external force model.

Here, the initial curve shown in Fig. 1a converges to the final curve shown in Fig. 1b. Although the final curve *does* satisfy (2), it is obviously an undesirable outcome. The external force used in this example is given by $-\nabla E_{\text{ext}}$, where $E_{\text{ext}} = -|\nabla(G_\sigma * I)|^2$, the *edge energy functional* defined in [1]. Here, G_σ is a Gaussian blur with standard deviation σ. Shown in Fig. 2a, this external force tends to pull an active contour apart within a concave region, which explains the result in Fig. 1b.

In this paper, we introduce an additional external force to the left hand side of (2) which will encourage the snake to move deeper into concave boundary regions as it evolves towards its equilibrium state.

2. A NEW EXTERNAL FORCE

Three guiding principles led our development of a new external force. First, we wanted to be able to add the new force to the existing one in a complementary fashion. Since the existing force is the gradient of a scalar function (the energy E_{ext}), it is an irrotational (curl-free) field. According to the Helmholtz theorem, the other fundamental field component is a solenoidal (divergence-free) field. Therefore, our new field should be solenoidal. Second, in the absence of internal forces the new field should not disturb the equilibrium contours of the external energy. Therefore, the new field should be zero whenever the field $-\nabla E_{\text{ext}}$ is zero. Third, we wanted this field to point toward the apex of concave boundary regions, a feature defined by the object boundary

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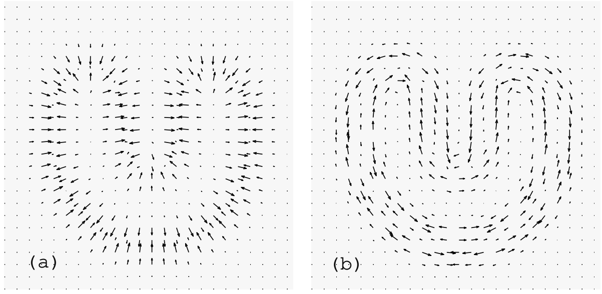

Figure 2: (a) External force $\mathbf{f}_g$ and (b) solenoidal field $\mathbf{f}_s$.

curvature. Therefore, our new field should use a measure of boundary curvature in its definition.

Let $\mathbf{f}_g = -\nabla E_{\text{ext}}$ and $\mathbf{f}_g^{\perp}$ be the 90° clockwise rotation of $\mathbf{f}_g$. It is easy to show that $\mathbf{f}_g^{\perp}$ is a solenoidal field. Further, with reference to Fig. 2a it is easy to see that these vectors point along the boundary, some pointing into the concave region and some pointing out. A simple measure of local object curvature allows us to distinguish these vectors and to reverse those pointing the wrong way.

To measure the local boundary curvature, we form a blurred image $I_b = G_\sigma * I$. The curvature of the level sets of I_b gives a rough estimate of the local boundary curvature. This curvature is given by

$$\kappa_l = \frac{{f_2}^2 f_{11} + {f_1}^2 f_{22} - 2 f_1 f_2 f_{12}}{({f_1}^2 + {f_2}^2)^{\frac{3}{2}}} \tag{3}$$

where $f_1, f_2, f_{11}, f_{12}, f_{21}, f_{22}$ are the first and second partial derivatives of I_b. The curvature map κ_l is positive near convex contours and negative near concave contours. It is easy to see that the vector field $-\nabla\kappa_l$ points from convex to concave boundary regions. Although this property is desirable, we do not use $-\nabla\kappa_l$ directly because: 1) it is an irrotational field, and 2) its sinks and sources are not positioned well. Instead we use it to flip the vectors in $\mathbf{f}_g^{\perp}$ when they are pointed the wrong way. To do this we define $\phi = \text{sgn}(< \mathbf{f}^{\perp}, -\nabla\kappa_l >)$ where $\mathbf{f}^{\perp}$ is the 90° counter-clockwise rotation of ∇I_b. ϕ is +1 when these vectors are aligned within ±90° and -1 when they form an obtuse angle. Our final solenoidal field is then given by

$$\mathbf{f}_s = \phi \mathbf{f}_g^{\perp} \tag{4}$$

It can be shown that this field is still solenoidal, although it may now be discontinuous. Fig. 2b shows $\mathbf{f}_s$ for the example of Fig. 1.

Finally, we construct the total external force $\mathbf{f}_t$ as convex combination of $\mathbf{f}_g$ and $\mathbf{f}_s$

$$\mathbf{f}_t = \tau \mathbf{f}_g + (1 - \tau)\mathbf{f}_s$$

where $\tau \in [0, 1]$. Using $\mathbf{f}_t$ in place of $-\nabla E_{\text{ext}}$ in (2) gives a new snake model which can be solved in the usual iterative fashion [1].

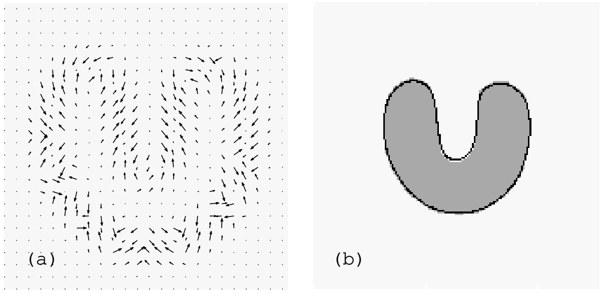

Figure 3: (a) Total field $\mathbf{f}_t$ and (b) final snake.

3. EXPERIMENTAL RESULT

Using $\tau = 0.5$ and $\sigma = 6$ pixels, the new external field for the example of Fig. 1 was computed. The result is shown in Fig. 3a. One can see that the new vector field should pull the snake both down into the concave region and toward the boundary. If the active contour were to be initialized inside the object, it would have the same effect except that the two lobes of the "U" are "concave" from the inside perspective. Using the same initial contour and parameters that led to the result in Fig. 1b ($\alpha = 0.5$, $\beta = 0.0$) the snake algorithm was run to convergence, leading to the final curve shown in Fig. 3b — a nearly perfect result.

4. CONCLUSION

We have proposed a new external force model for snakes. This new force is more general than the traditional external force since it incorporates a solenoidal field in addition to an irrotational field. It is calculated using local curvature information from the object boundary. Finally, we have demonstrated improved convergence on a simple noise-free example with a concave boundary region.

5. REFERENCES

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