ACCELERATION OF HIGH ANGULAR AND SPATIAL RESOLUTION DIFFUSION IMAGING USING COMPRESSED SENSING

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ABSTRACT

Achieving simultaneously high angular and spatial resolution in diffusion imaging is challenging because of the long acquisition times involved. We propose a novel compressed sensing method to acquire high angular and spatial resolution diffusion imaging data, while keeping the scan time reasonable. We show that joint under sampling of 6-D k-q space is more efficient than undersampling only one of the dimensions. We use a sparse Gaussian mixture model and an iterative reconstruction scheme to recover the peaks of the orientation distribution functions (ODF) with high accuracy. We show that at least 6-fold acceleration of acquisition is possible, thereby enabling high angular and spatial resolution diffusion imaging in a reasonable scan time.

Index Terms— Compressed sensing, Orientation distribution function, high resolution

1. INTRODUCTION

Both high angular and spatial resolution are desirable for accurate fiber tractography applications of diffusion imaging. High angular resolution is required to accurately detect crossing fiber architectures that co-exists in voxels [1]. Similarly, high spatial resolution acquisition minimizes confounds such as partial volume effects, thus improving the accuracy of the parameters. The main challenge with high angular and spatial resolution diffusion imaging is the unreasonably long scan time. Specifically, the acquisition time increases linearly with the number of diffusion directions. The increase in k-space extent to achieve high spatial resolution translates to long readouts with single shot sequences, thus making the acquisition more sensitive to field inhomogeneity and T_2^* distortions. It is a common practice to use multi-shot sequences to minimize these distortions, which again results in increased scan time. Due to these challenges, the usual practice is to enhance the resolution of the preferred dimension, while compromising the other.

Several methods have been suggested to accelerate diffusion imaging, which can be broadly classified into two categories: uniform k-only down-sampling schemes and uniform q-only down-sampling schemes (see Fig 1 below). Several researchers achieved high spatial resolution of sub-millimeter scales within reasonable scan times using parallel imaging and under-sampling in k-space [2][3]; however the angular resolution was restricted in these studies. Similarly, using compressed sensing and under-sampling in the q-space, a few groups have recently achieved high angular resolution in a reasonable scan time [4, 5], however at low spatial resolution. A third approach was proposed by [6],[7] where high spatial and angular resolution was achieved. However, to reduce scan time, the spatial coverage was compromised to a limited field of view (FOV).

In this work, we propose a new acquisition strategy that can simultaneously enhance the spatial and angular resolutions without compromising on the FOV, while keeping the scan time reasonable. Our approach also makes use of the theory of compressed sensing. However, as opposed to the previous methods where down-sampling was restricted to either the k-space or the q-space, we propose to jointly downsample the combined k-q space. We use a multi-shot variable density spiral trajectory similar to earlier schemes that acquire high spatial resolution data [3]. However, we only acquire a subset of the spatial interleaves for each shot. Specifically, we under sample the k-space corresponding to each direction randomly. Since we are jointly under sampling the k-q space, we expect this scheme to provide improved reconstructions, compared to schemes that only under sample one of the directions. We model the multi-modal diffusion signal in each voxel as a sparse linear combination of Gaussian basis functions; this Gaussian mixture model is widely accepted in the diffusion community [8] and is discussed in more detail in the text. We pose the recovery of the diffusion signal from all the voxels of the dataset as a single regularized reconstruction problem. We use ℓ_1 penalty on the coefficients of the basis functions to enhance sparsity, while we use a total variation penalty on the coefficients to exploit the spatial smoothness. Note that this approach is more robust than a voxel-by-voxel fitting pursued by other researchers[5]. We use an iterative re-weighted conjugate gradient (CG) algorithm to solve the above reconstruction problem.

We demonstrate the utility of the proposed scheme by retrospectively downsampling a fully sampled 60 direction spiral diffusion weighted dataset with 1 mm^2 in-plane resolution. Our results show that the proposed scheme can considerably reduce the reconstruction errors, compared to q-only down-

sampling. We demonstrate a six fold speedup with minimal errors.

2. BACKGROUND

The measured signal, S, from a diffusion weighted sequence can be written as a 6-D Fourier Transform of the diffusion displacement probability density function (PDF) P, as:

$$S(k,\mathbf{q},\tau) = S_0 \int_{\mathbf{x}^3}\int_{\mathbf{r}^3} P(\mathbf{x},\mathbf{r},\tau)e^{-2\pi i(\mathbf{k}^T\mathbf{x}+\mathbf{q}^T\mathbf{r})}d\mathbf{x}d\mathbf{r} \tag{1}$$

where S_0 is the reference image, $\mathbf{x}$ is the spatial vector, $\mathbf{r}$ is the diffusion displacement vector and $\{\mathbf{x},\mathbf{r} \in \mathcal{R}^3\}$. τ is the time for diffusion, $\mathbf{k}$ and $\mathbf{q}$ are the Fourier duals of $\mathbf{x}$ and $\mathbf{r}$ vectors respectively.

It is clear from (1) that the acquisition space is a 6-D space spanned by the k-q space, the dual space of diffusion-physical space. In order to reconstruct the PDF accurately, one needs to sample the combined k-q space adequately, satisfying the Nyquist sampling criteria. Because of the high sampling burden involved, reconstruction of PDF is almost never done in practice. Instead, a derived measure, the diffusion orientation distribution function (ODF) is used for tractography applications. ODF is the defined as the radial projection of PDF on to a unit sphere:

$$\Phi(\mathbf{u}) = \frac{1}{Z}\int_0^\infty P(\mathbf{u}r)d(r) \tag{2}$$

for direction $\mathbf{u}$ where Z is a dimensionless normalization constant. ODFs are typically computed analytically for different diffusion models rather than using (2).

3. THEORY

The theory of compressed sensing (CS) allows one to reconstruct a band-limited signal from sub-Nyquist sampled data, provided the signal is sparse. We use the above theory to reconstruct the sparse diffusion directions in each voxels from a sub-Nyquist sampled data. CS requires three components: an under-sampling strategy that can incoherently sample the k-q space, a sparse model that can represent the multi-modal diffusion and a reconstruction scheme that can recover the diffusion peaks without aliasing artifacts. We explain each of the three components that we used in our work below:

3.1. Joint k-q under-sampling

Previous under-sampling strategies used in diffusion imaging can be broadly classified into two categories: uniform k-only under-sampling strategies (eg: parallel imaging) and uniform q-only under-sampling strategies (eg: collecting few angular samples, but reconstructing at high angular resolution). These schemes decoupled the combined k-q acquisition space and performed down-sampling in only one domain. For simultaneous improvement of resolution in spatial and angular domains, under-sampling in one dimension alone does not provide enough acceleration. We propose a joint under-sampling of the combined k-q space as follows: the object is sampled using relatively high number of diffusion directions, however the spatial domain of each diffusion direction is randomly under-sampled. This approach is aimed at collecting high resolution information from both domains, at the same time reducing redundancy incoherently. We achieved this by using a multi-shot variable density spiral trajectory and skipping multiple random inter-leaves of the spiral when sampling each diffusion direction. Significant savings in acquisition time can be achieved using this scheme. A pictorial representation of the various acquisition strategies is shown in Fig 1.

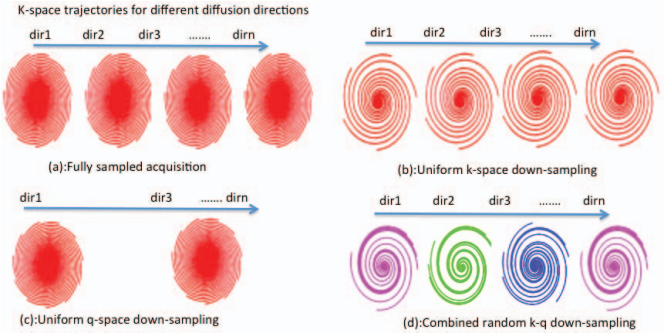


Fig. 1. Various under-sampling strategies

3.2. Sparse Model for compressed sensing recovery

Typical voxel sizes in MRI are 2-3 orders of magnitude larger than the underlying axonal structures. To represent the heterogeneity in the orientation of these structures within a voxel using diffusion measurements, Gaussian mixture models (GMM) have been introduced in the past. A discrete diffusion model based on the GMM was extensively studied in [8] where a set of diffusion basis functions $\{\psi\}$, were generated from a tensorial basis. We use this model to represent the diffusion signal in each voxel sparsely. N basis tensors distributed uniformly in the q-space are derived from a base tensor D which was chosen based on some realistic assumptions. For the human brain, it is reasonable to assume a diffusivity value of $1e^{-3}mm^2/s$ in the white matter voxels and expect the longitudinal fiber diffusion to be about five times the transversal diffusion. Based on this a base tensor $\mathbf{D} = 1e^{-3} * [1700\ 0\ 0; 0\ 300\ 0; 0\ 0\ 300]$ was formed. The N basis tensors were obtained by rotating $\mathbf{D}$ to N unit vectors uniformly distributed in the 3-D q-space such that $\mathbf{D}_i = \mathbf{R}_i * \mathbf{D} * \mathbf{R}_i{}^T; i = 1 : N$ where $\mathbf{R}_i$ is the rotation matrix for the i^{th} direction. We define the basis functions $\psi_i(b,\mathbf{g}) = e^{-b\mathbf{g}^T\mathbf{D}_i\mathbf{g}}$. Then,

$$\frac{S(b,\mathbf{g})}{S_0} = \sum_{i=1}^{N} f_i \,.\psi_i(b,\mathbf{g}) \tag{3}$$

where $S(b, \mathbf{g})$ is the diffusion signal measured using the diffusion sensitizing parameter b and diffusion gradient orientation $\mathbf{g}$ and S_0 is the attenuation-free non-diffusion-weighted image. With the tensors $\mathbf{D}_i's$, being already defined, the coefficients of the tensorial basis functions, $f_i's$, are the unknowns in the above model. In a given voxel, fibers oriented in all possible directions cannot be present at the same time. When fibers in some orientations are not present in a voxel, the coefficients of the corresponding basis functions would be zero. Thus, one can see that the f's will be sparse in a given voxel.

3.3. Joint Reconstruction of $\mathbf{f}$ from k-q undersampled data

The N unknowns in each voxel can be jointly reconstructed from the under-sampled k-q data using compressed sensing. For the applied gradient directions $\mathbf{g_k}$ and the applied b-value, (3) results in a set of K linear equations ($K < N$) which can be written in matrix formulation as follows: Define $y_k = \mathcal{F}\{S_k/S_0\}$, where $\mathcal{F}$ is the Fourier Transform. y_k is the measured undersampled data corresponding to direction g_k. Then (3) can be re-written as $\mathbf{y} = \hat{\mathbf{A}}\mathbf{f} + \epsilon$ where $\mathbf{A} =$

$$\begin{bmatrix} \psi_1(b, \mathbf{g_1}) & \cdots & \psi_N(b, \mathbf{g_1}) \\ \vdots & \vdots & \vdots \\ \psi_1(b, \mathbf{g_K}) & \cdots & \psi_N(b, \mathbf{g_K}) \end{bmatrix} \text{ and } \hat{\mathbf{A}}\mathbf{f} = \mathcal{F} \circ \mathbf{A}(\mathbf{f})$$

Since the k-space data was collected on a non-Cartesian grid, the Fourier transform $\mathcal{F}$ represents encodings to the non-Cartesian k-space locations. The unknown vector $\mathbf{f}$ can now be solved as a non-linear optimization problem as posed in (4) that enforce sparsity of $\mathbf{f}$ by minimizing the L1 norm of $\mathbf{f}$. Additional regularization using as total variation is also imposed to constrain the solution.

$$\begin{aligned} \hat{\mathbf{f}} &= \underset{f}{\operatorname{argmin}} \, ||\hat{\mathbf{A}}\mathbf{f} - \mathbf{y}||_{l2}^2 + \lambda_1 ||\mathbf{f}||_{l_{TV}} + \lambda_2 ||\mathbf{f}||_{l1} \\ &\text{where } ||\mathbf{f}||_{l_{TV}} = ||\nabla \mathbf{f}||_{l1} \text{ and } ||\mathbf{f}||_{l1} = \sum_i |\mathbf{f}_i| \end{aligned} \tag{4}$$

We minimize the above cost function using a CG algorithm. The ℓ_1 minimization was implemented as an iterative re-weighted ℓ_2 norm minimization. Once $\mathbf{f}$ is solved, the diffusion images can be reconstructed using the forward model in (3). The diffusion ODF can also be computed for the above model using the analytical expression [9]:

$$\Phi(b, \mathbf{u_k}) = \sum_{i=1}^{N} \frac{f_i}{Z} \sqrt{\frac{\pi b}{\mathbf{u_k}^T \mathbf{D}_i^{-1} \mathbf{u_k}}} \tag{5}$$

4. RESULTS

A numerical phantom simulating a 90-degree crossing fiber and an in-vivo data were used to test the algorithm. In-vivo human data at high angular and spatial resolution was collected and retrospectively down-sampled. Images of a healthy adult volunteer were obtained on a 3T GE MR750 scanner (GE Healthcare, Waukesha, MI) equipped with an 8-channel head coil, after approval by Duke University Health System Institutional Review Board. Scanning parameters: variable density spiral sequence: FOV 19.2cm, matrix 192x192, 1x1mm^2 resolution, 10 slices, slice thickness/gap = 1.5mm/2.0mm, b=1200 s/mm^2, 5 b=0 and 60 diffusion-weighted images, 22 interleaves, TE/TR=40/2000ms, total scan time was 48 mins.

We accommodate the data from multiple channels using SENSE scheme. To test the performance of the reconstruction, we performed experiments at various under-sampling levels. In addition to the joint k-q downsampling, a uniform q-only downsampling as in [4, 5] was also performed retrospectively for comparisons.

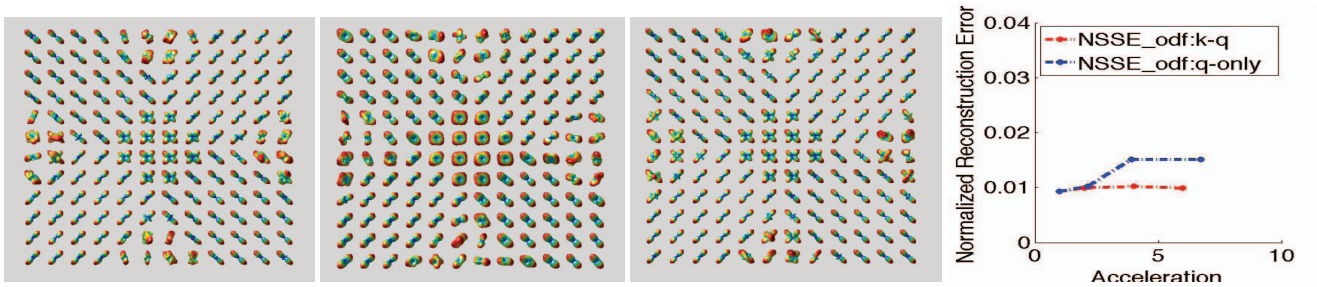
For the in-vivo data, a reference ODF was reconstructed from the fully sampled data with all 60 directions and 22 interleaves. We achieved the combined k-q down-sampling as follows: for an acceleration factor of n, 22/n random interleaves were chosen for each diffusion direction such that the selected interleaves differ for each diffusion direction. Acceleration rates corresponding to n=2,4,6 (corresponding to 11, 5, 3 interleaves) were tested. The same acceleration rates corresponding to n=2,4,6 (corresponding to 30, 15 and 10 diffusion directions) were tested for q-only down-sampling scheme also. The ODFs from both under-sampling schemes were reconstructed and compared to the reference ODF. A normalized sum-of-squares error (NSSE) was computed for the reconstructed ODFs in each case as follows:

$$NSSE = \frac{\sum_{x,y,z,K} \left(||ODF_{Ref} - ODF_{Rec}||^2 \right)}{\sum_{x,y,z,K} ||ODF_{Ref}||^2} \tag{6}$$

A plot of the error in the reconstructed ODF from the two under-sampling schemes at various levels of acceleration are shown in Fig 2(d) and Fig 3(d). The error in the k-q down-sampling scheme is much lower than that of the q down-sampling scheme for all acceleration factors. The results shows that to achieve the same acceleration rate, the combined k-q down-sampling scheme performed better than q-only down-sampling. From the reconstructed ODFs of the two schemes (Figures 2 a-c,3 a-c), it can be seen that the ODF peaks becomes less pronounced as the q down-sampling rate is increased, whereas the peaks are still preserved accurately in the k-q down-sampling schemes for the same acceleration rates. Results show that even at 6-fold acceleration of the k-q down-sampling scheme, the diffusion ODF can be reconstructed reasonably accurately.

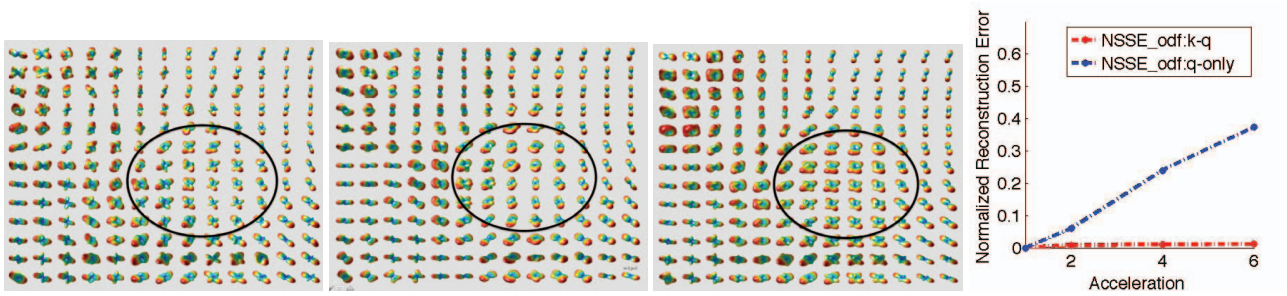
5. CONCLUSION

In spite of the obvious benefits of combining high angular and spatial resolution, prohibitively long acquisition times



(a) (b) (c) (d)

Fig. 2. Numerical phantom results: (a) Reference ODF reconstructed using 47 angular measurements. (b),(c) ODF reconstructed at acceleration of 4 using q-only and k-q down-sampling respectively. (d) Plot of Normalized reconstructed error for ODF for the two schemes at various accelerations .



(a) (b) (c) (d)

Fig. 3. Real data results: (a) Reference ODF reconstructed from a region with 3 fiber crossings. (b),(c) ODF reconstructed at acceleration of 4 using q-only and k-q down-sampling respectively (d) Plot of Normalized reconstructed error for ODF for the two schemes at various accelerations.

have made such goals impractical with conventional diffusion imaging schemes. We proposed a method that can achieve both high spatial and angular resolution simultaneously within a reasonable scan-time. We have shown that the combined k-q acquisition space of diffusion can be substantially down-sampled and diffusion ODFs accurately reconstructed using compressed sensing. A sparse model was formulated and a novel acquisition strategy was introduced that can down-sample the combined k-q space. Results on numerical phantom and real data shows that to achieve the same factor of acceleration, combined k-q down-sampling scheme is better than uniform q-only down-sampling to preserve the directional accuracy of crossing fibers. Our results show that the acquisition time can be reduced by at least 6-fold by appropriately under-sampling the combined k-q acquisition space.

6. REFERENCES

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