THREE-DIMENSIONAL HYBRID-ENCODED MRI USING COMPRESSED SENSING

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ABSTRACT

Compressed sensing (CS) has been successfully applied to accelerate conventional magnetic resonance imaging (MRI) with Fourier encoding. However in Fourier encoding, the low spatial frequency always has to be fully sampled such that the high frequency is insufficiently sampled at high accelerations, leading to serious loss of resolution. Non-Fourier encoding has been studied in the context of compressed sensing for 2-D imaging, but with limited improvement. In this paper, we propose a novel 3-D acquisition method using hybrid encoding. The method exploits random encoding with a circulant structure along the "phase encoding" direction, while keeping conventional Fourier encoding along the readout and slice encoding directions. A 2-D random undersampling pattern is used for accelerating the scans. Both simulation and experimental results demonstrate that the proposed method preserves much better resolution than the conventional 3-D Fourier encoding when the same acceleration factors are used.

Index Terms— Compressed Sensing, magnetic resonance imaging, non-Fourier encoding, circulant random encoding, hybrid encoding

1. INTRODUCTION

Compressed Sensing (CS) has emerged as a new sampling theory which allows sparse or compressible signals to be sampled at a rate that is close to their intrinsic information rate and well below their Nyquist rate [1,2]. Such reduced sampling is very desirable in MRI because the data acquisition speed is directly related to the amount of data that needs to be acquired. CS has been successfully used in conventional Fourier-encoded MRI for fast imaging [3,4]. In conventional MRI, the application of compressed sensing is made possible by the facts that (1) most MR images are compressible by certain transforms and (2) the desired image is Fourier encoded in the measurement (so called k-space) which allows incoherent sampling. To accelerate the imaging speed, the *k*-space is undersampled along the directions other than readouts. For example in 3-D Cartesian sampling, a variable density random sampling pattern is used to undersample the *k*-space data along the phase encoding and slice encoding directions [3]. The image is recovered by solving a constrained convex minimization problem. In SparseMRI [3], image $\mathbf{f}$ is given by

$$\arg\min_{\mathbf{f}} \{\left\|\mathbf{b}-\mathbf{F}_u\mathbf{f}\right\|_2^2 + \lambda_1 \left\|\mathbf{W}\mathbf{f}\right\|_1 + \lambda_2 \,\mathrm{TV}(\mathbf{f})\} \qquad (1)$$

where $\mathbf{b}$ is the measured *k*-space data; $\mathbf{F}_u$ is the random subset of the rows of the Fourier encoding matrix; $\mathbf{W}$ is the sparsifying transform matrix; TV(·) is total variation; λ_1 and λ_2 are constant regularization parameters.

Although CS has been demonstrated to accelerate the speed of Fourier encoded MRI in several imaging applications, Fourier encoding is known to have limitations [5,6] and may not be optimal for CS MRI. First, Fourier matrix is known to need more measurement than random matrices for exact recovery. Second, Fourier encoding does not spread out the signal energy for natural images, but concentrates the energy in the low-frequency region. Nyquist sampling is still required in the low-frequency region, which leads to insufficient sampling of high-frequency components at high undersampling factors. Finally, Fourier encoding is not universal. It is only maximum incoherent with the canonical basis or the fine scales of a wavelet transform. The incoherent condition is only weakly satisfied for some sparsifying transforms such as the coarse scale of wavelet transform [5].

A few works have investigated use of non-Fourier encoding for CS MRI. For example, independently identically distributed (i.i.d.) Gaussian random matrices have been proved to have several favorable properties as a CS matrix, such as universal and spread out. It has been studied for MRI as an alternative to Fourier encoding [7]. However, the memory usage and computations are demanding in reconstructing the large-scale 3D image volume. Toeplitz random matrices have been shown to share similar properties as i.i.d. random matrices, but with more efficient memory usage and computations. Toeplitz random encoding has also been investigated by the authors for CS MRI [8-10]. Although simulations show some promising results, a complete 2-D random excitation may not be practical because it requires a very long RF pulse. Spread spectrum MRI [11] is able to address this issue by replacing the random modulation by a linear chirp modulation, which is implemented physically using a second-order shim coil.

The linear chirp is able to spread out the signal energy in the measurement domain. However, none of these existing non-Fourier methods have demonstrated notable improvement over Fourier encoding using real experiments.

In this paper, we propose a hybrid encoding scheme for 3-D imaging. Specifically, based on the conventional 3-D Fourier imaging, we replace the Fourier encoding along the phase encoding direction by a circulant random encoding. The encoding scheme excites the object along the "phase encoding" direction randomly. This excitation profile is repeated but shifted by a single pixel spatially during each excitation. The corresponding sensing matrix has a circulant structure with independent random complex elements. The undersampling is carried out in a hybrid encoded 2-D space along both the slice encoding (Fourier) and circulant encoding directions. With the proposed hybrid encoding scheme, the benefits of both Fourier encoding and random encoding can be achieved simultaneously. The low frequency signal can still be densely sampled in the Fourier encoding direction for high SNR. At the same time, the signal energy is more spread out in the circulant encoding direction such that high-frequency information is adequately sampled. Using both simulation and real experiments, we demonstrate that hybrid encoding is superior to Fourier encoding in preserving resolution when the same reduction factor is used.

2. PROPOSED HYBRID ENCODING METHOD

2.1. Circulant random matrices

Although i.i.d random matrices have been proved to satisfy the restricted isometry property [2], the computation and memory usage are expensive for reconstructing MR images. Toeplitz and circulant matrices haven been shown to be almost as effective as the i.i.d. Gaussian random matrices in the context of CS, but require a significant reduction in computation and storage [12-16]. In particular, a circulant matrix is a special Toeplitz matrix and has the form

$$\mathbf{T} = \begin{pmatrix} h_1 & h_2 & \cdots & h_n \\ h_n & h_1 & h_2 & h_{n-1} \\ \vdots & \ddots & \ddots & \vdots \\ h_2 & h_3 & \cdots & h_1 \end{pmatrix} \qquad (2)$$

This matrix is known to be decomposed to $n^{-1/2}\mathbf{F}_1^* \sum \mathbf{F}_1$, where matrix $\mathbf{F_1}$ is the 1-D Fourier matrix, and $\sum = diag\left(\left[\sigma_1, \sigma_2, \cdots \sigma_n\right]\right)$ is a diagonal matrix whose non-zero entries are the 1-D Fourier transform of **h**. Such a decomposition suggests that multiplication of the circulant matrix **T** with a signal **x** (equivalent to circular convolution) can be efficiently calculated by a point-wise production of two length-*n* vectors in the Fourier domain. A fast algorithm for CS reconstruction has also been developed for circulant measurements based on this property. In our design, σ_i's are chosen as complex numbers with uniform magnitudes and random phases which is uniformly distributed on $[0, 2\pi]$. Such a choice can further reduce computation.

2.2. Hybrid encoding scheme

Our objective is to design an MR physical-implementable encoding scheme for 3-D imaging that is superior to the conventional Fourier encoding in the CS framework. We have demonstrated the feasibility of circulant random encoding in 2-D imaging using MR experiments in our prior work [9]. In this work, we integrate circulant random encoding with conventional Fourier encoding in 3-D imaging to achieve 2-D accelerations. The proposed method exploits random encoding with a circulant structure along the "phase encoding" direction, while keeping conventional Fourier encoding along the readout and slice encoding directions. Figure 1 illustrates the 3-D imaging technique. Without loss of generality, x is assumed to be the readout, y the circulant encoding, and z the slice encoding. The undersampling is done in the k_y-k_z plane of *k*-space, where a variable density random sampling is used along the k_z direction (Fourier encoding), and a uniform random sampling is along the k_y direction (circulant encoding).

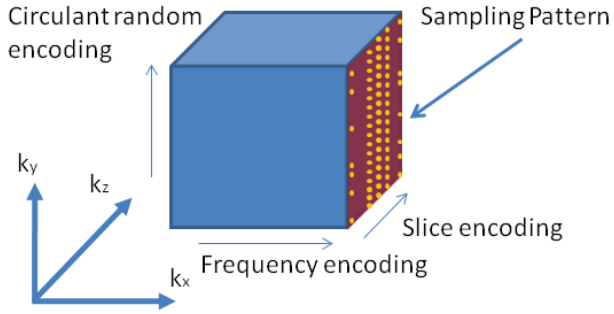


Fig.1 Illustration of the proposed hybrid encoding method in 3-D acquisition.

The image is reconstructed in the y-z plane for each x. Assuming the image vector to be reconstructed in the y-z plane as **f**, the encoding matrix **H** can be written as the Kronecker product of a circulant random matrix (i.e., **T** in Eq. (2)) and a 1-D Fourier transform matrix. The matrix **H** has a block circulant structure with each block being a Fourier transform matrix. The original image vector **f,** can be reconstructed from the undersampled data **b** by solving

$$\arg\min_{\mathbf{f}} \{\left\|\mathbf{b} - \mathbf{H}_u\mathbf{f}\right\|_2^2 + \lambda_1 \left\|\mathbf{W}\mathbf{f}\right\|_1 + \lambda_2 \,\mathrm{TV}(\mathbf{f})\} \qquad (3)$$

where all notations are the same as in Eq. (1) except that the Fourier matrix **F** is replaced by the circulant-Fourier encoding matrix **H**.

2.3. Pulse sequence implementation

The pulse sequence designed for the proposed circulant random encoding is shown in Fig. 2. Specifically, a random

RF pulse with a constant magnitude and random phases is applied when a fixed gradient Gy is on to generate random flip angles along the y direction.

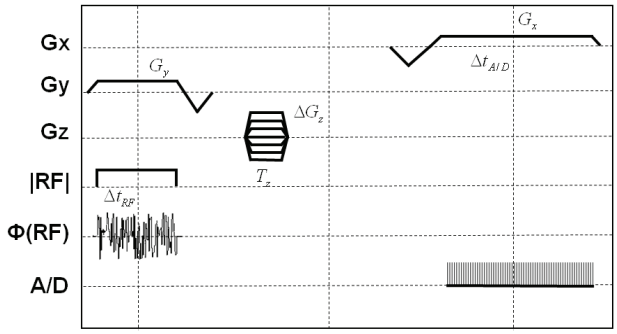


Fig. 2 Timing diagram of the designed pulse sequence for the proposed 3-D hybrid encoding scheme.

According to the MR physics of magnetization with small tip angles, the excitation profile is [17]

$$M(\mathbf{r}) = j\gamma M_0(\mathbf{r})\int_0^T B_1(t)e^{j\mathbf{k}(t)\cdot\mathbf{r}}\cdot dt \qquad (4)$$

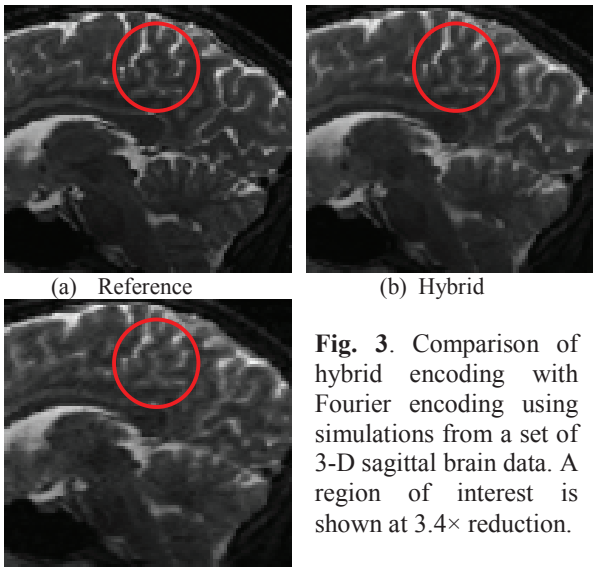
which depends on the Fourier transform of the random pulse $B_1(t)$. Each random RF pulse generates a random excitation profile along the y direction. To construct the circulant structure for the random encoding, a linear phase shift is applied to the RF pulse at subsequent excitations. The linear phase shift is designed such that the excitation profiles from consecutive excitations are spatially shifted by a single pixel along the y direction. Each excitation corresponds to a row of the circulant encoding matrix **T** in Eq. (2). After the RF excitation, the slice encoding gradient Gz is turned on along z followed by the readout gradient Gx along x. For an isotropic field of view (FOV), the gradients are designed to satisfy,

$$G_y\Delta t_{RF} = G_x\Delta t_{A/D} = \Delta G_z T_z \qquad (5)$$

where Δt_{RF} and $\Delta t_{A/D}$ are the RF excitation and A/D acquisition sampling periods, and T_z is the duration of Gz gradient.

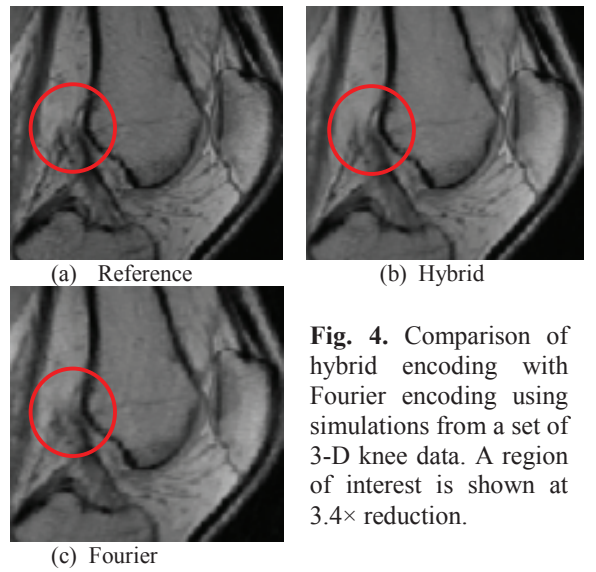
3. RESULTS

We conducted both simulations and experiments to demonstrate the performance of the proposed hybrid encoding method. The results were compared with those of the conventional Fourier encoding in the context of CS. For Fourier encoding, we followed the sampling scheme used in [3], where a 2-D variable density random undersampling was used to sample the low spatial frequency region densely. Non-linear conjugate gradient [3] was used to reconstruct the image based on Eq. (1) for Fourier and Eq. (3) for hybrid encoding. Both wavelet and TV were used to sparsify the image. In simulation, two sets of 3-D data (sagittal brain and knee) were used to generate the simulated data acquired through hybrid encoding. Both images are of 256×256×16. The 2-D undersampling was done in the ky-kz plane from the full size of 256×16. A reduction factor of 3.4 was used. Figures 3 and 4 compare the reconstructed images in x-y plane for the brain and knee datasets respectively. The hybrid encoding method is seen to preserve better resolution than Fourier encoding for both datasets (particularly indicated in red circles).



(a) Reference (b) Hybrid (c) Fourier

Fig. 3. Comparison of hybrid encoding with Fourier encoding using simulations from a set of 3-D sagittal brain data. A region of interest is shown at 3.4× reduction.



(a) Reference (b) Hybrid (c) Fourier

Fig. 4. Comparison of hybrid encoding with Fourier encoding using simulations from a set of 3-D knee data. A region of interest is shown at 3.4× reduction.

In actual experiments, a pummelo was scanned on a 3T commercial MRI scanner (GE Healthcare, Waukesha, WI) with one standard head coil using both the 3-D hybrid encoding pulse sequence (RF pulse duration: 6.144 ms) shown in Fig. 2 and the conventional 3-D gradient echo pulse sequence (both with TE/TR: 15/40ms; rBW: 5.96kHz; FOV: 15cm^2; matrix: 256×256×32; Flip angle: 7°). The data

were acquired with full Nyquist sampling and then manually undersampled by removing some data points based on the designed sampling patterns. The 2-D undersampling was done in the ky-kz plane from the full size of 256×32. A reduction factor of 2.0 was used. The images reconstructed from the full data were used as the reference for comparison. Figure 5 shows comparison between the hybrid encoding and Fourier encoding. It is seen that the experimental results agree with the conclusion from simulations that the proposed hybrid encoding preserves more details and sharp edges than Fourier encoding.

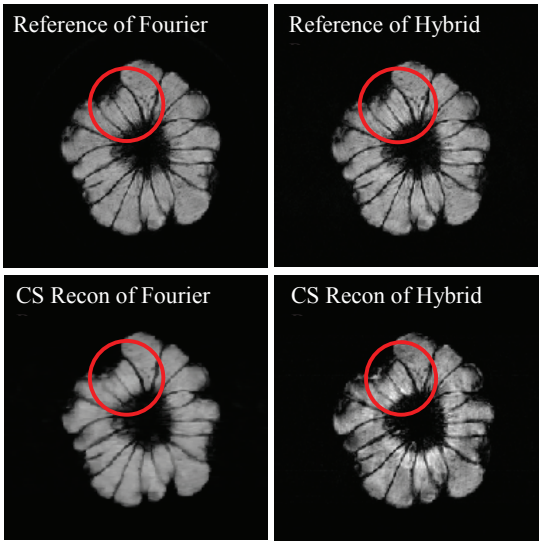


Fig. 5. Comparison of 3-D hybrid encoding with Fourier encoding using actual experiment on a pummelo. A region of interest is shown at 2.0× reduction.

4. DISCUSSION AND CONCLUSION

In this paper, we propose a novel 3-D hybrid random encoding scheme which integrates the benefits of both random encoding and Fourier encoding. The Fourier encoding with variable density sampling ensures signals with high SNR to be sampled densely, while random encoding spreads out the signal energy to allow high spatial frequency to be sampled adequately for preserving resolution. The circulant structure for random encoding also can allow efficient reconstruction algorithms. The computational complexity of CS reconstruction is about the same for both hybrid and Fourier encodings. Both simulation and real experiment show that the proposed hybrid encoding scheme outperforms Fourier encoding in preserving resolutions.

5. ACKNOWLEDGMENT

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6. REFERENCES

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