# K-T CSPI: A DYNAMIC MRI RECONSTRUCTION FRAMEWORK FOR COMBINING COMPRESSED SENSING AND PARALLEL IMAGING

*Yihang Zhou[1], Yuchou Chang[2], Dong Liang[2, 3], Leslie Ying[1, 2]*

[1]Department of Electrical Engineering, State University of New York at Buffalo, Buffalo, NY, USA
[2]Department of Electrical Engineering and Computer Science,
University of Wisconsin-Milwaukee, Milwaukee, WI, USA;
[3]Shenzhen Institutes of Advanced Technology, Chinese Academy of Sciences, Shenzhen, China.

## ABSTRACT

In this paper, we propose a new dynamic MR image reconstruction technique that combines the compressed sensing-based dynamic methods with parallel imaging techniques to achieve high accelerations. The method decouples the reconstruction process into two sequential steps. In the first step, a series of aliased dynamic images is reconstructed using a CS method from the highly undersampled *k*-space data. In the second step, the missing *k*-space data for the original image are reconstructed by the nonlinear GRAPPA technique. The sampling strategy for each step is thereby designed independently such that the incoherent undersampling requirement for CS and structured undersampling requirement for parallel imaging can be satisfied simultaneously. Experimental results demonstrate that the proposed method improves the reconstruction quality of dynamic cardiac cine MRI over the state-of-the-art method.

***Index Terms***— dynamic MRI, compressed sensing, nonlinear GRAPPA, spatial-temporal total variation

## 1. INTRODUCTION

Dynamic magnetic resonance imaging (MRI) is a technique to acquire a time series of images from a moving object at a high frame rate. In dynamic MRI, the *k*-space measurements are made at different time. To obtain an artifact-free image series using conventional Fourier reconstruction, the Nyquist sampling criterion has to be satisfied in both spatial and temporal directions. However, due to the low data acquisition speed in dynamic MRI, this criterion is difficult to satisfy, which results in aliasing artifacts or motion artifacts in the conventional Fourier reconstruction. Fast acquisition speed is important for high spatial-temporal resolution. A range of techniques have been developed to reconstruct a high-quality image series from the undersampled MRI data by exploiting spatial and/or temporal correlations in the dynamic image series. These spatial and temporal correlations render it feasible to estimate the missing data from undersampled measurements. Typical methods for the reconstruction of undersampled single-coil measurements include RIGR [1], keyhole [2], view-sharing [3], UNFOLD [4], Partially Separable Function (PS) [5,6], Kalman filter [7], PARADIGM [8], and *k-t* BLAST [9].

Among the fast imaging techniques, parallel imaging and compressed sensing are two promising techniques that accelerate the acquisition speed through undersampling the *k*-space below the Nyquist rate. Parallel imaging [10,11] reconstructs the missing *k*-space data using the information that signals received from multiple coils have distinct spatial sensitivities. Compressed sensing (CS) [12-14] exploits the sparseness of the image in a certain domain to obtain the original image. Due to the complementary information used in these two techniques, it is of great interest to combine these two techniques. Several techniques [15-17] have been developed to combine pMRI and CS for static imaging. However, in most CS-based dynamic imaging methods [18-22], the parallel imaging capability is simply incorporated by replacing Fourier encoding by sensitivity encoding in the data consistency term. Like Sparse SENSE [15], the counterpart for static imaging, such a simple combination is difficult to maximize the accelerations that can be achieved by each individual method because each method has different sampling requirements. In addition, sensitivities estimated from composite images can be inaccurate and thereby can degrade the image quality.

In this paper, a new dynamic MRI reconstruction scheme is proposed that combines the CS-based dynamic imaging method and parallel imaging technique more effectively. The method decouples the reconstruction process into two sequential steps. In the first step, a series of aliased dynamic images is reconstructed using a CS method from the highly undersampled *k*-space data. In the second step, the missing *k*-space data for the original image are reconstructed by the nonlinear GRAPPA technique. The sampling strategy for each step is thereby designed independently such that the incoherent undersampling requirement for CS and structured

undersampling requirement for parallel imaging can be satisfied simultaneously. The performance of the proposed method is evaluated using *in vivo* MR cardiac cine experiments.

## 2. PROPOSED METHOD

### 2.1 Data acquisition

In CS, incoherent sampling is required for reconstruction from undersampled data. In CS-based dynamic imaging, a variable density random pattern is usually used to undersample the *k-t* space. Specifically, for each time frame *t*, the *k*-space is undersampled by taking a random subset of the full *k*-space data with the central *k*-space sampled more densely. Different random patterns are used at different time *t*. On the other hand, parallel imaging usually desires a set of uniformly undersampled *k*-space data, especially for GRAPPA based methods. Such uniform undersampling can simplify computation significantly.

The contradictory on sampling requirements motivates us to decouple the CS and parallel imaging into independent steps. Specially, the *k-t* space is undersampled by taking a random subset of the already uniformly undersampled *k*-space data at each time point. The uniform undersampling is for parallel imaging with a reduction factor of $R_1$, and the random subset is for CS requirement which achieves an additional reduction factor of $R_2$. The combined reduction is thus $R_1 \times R_2$. Extra auto calibration signals (ACS) are also acquired at the central *k*-space for all time frames.

### 2.2. Image Reconstruction

We use $\boldsymbol{M}_u$ to represent the uniform undersampling operation, and $\boldsymbol{M}_r$ for taking a random subset in *k*-space. The imaging equation for the proposed method is given by:

$$\mathbf{d}_{\mathrm{u,r}} = \boldsymbol{M}_r \boldsymbol{M}_u \mathbf{d}_{\mathrm{full}}\,, \qquad (1)$$

where $\mathbf{d}_{\mathrm{u,r}}$ is the acquired data in *k-t* space, $\mathbf{d}_{\mathrm{full}}$ is the desired full data also in *k-t* space. According to the decoupled undersampling operations $\boldsymbol{M}_u$ and $\boldsymbol{M}_r$, the reconstruction is carried out in two sequential steps. In the first step, the goal is to reconstruct the uniformly undersampled *k*-space by "inverting" the operation $\boldsymbol{M}_r$. We use a CS-based algorithm to reconstruct a series of aliased images over time (*x-t* domain). Fourier transform is then applied on each aliased image at different time to generate a series of *k*-space data (*k-t* space). These data correspond to the uniformly undersampled data in the full *k*-space. In the second step, nonlinear GRAPPA is used frame by frame to recover the missing *k*-space data by "inverting" the operation $\boldsymbol{M}_u$. Figure 1 illustrates the reconstruction procedures of the proposed algorithm. A detailed explanation of each step is given below

#### 2.2.1 Step One: CS Reconstruction with Spatial and Temporal Total Variation Regularization

Our goal in this step is to reconstruct a series of aliased images from undersampled *k*-space data. CS reconstruction

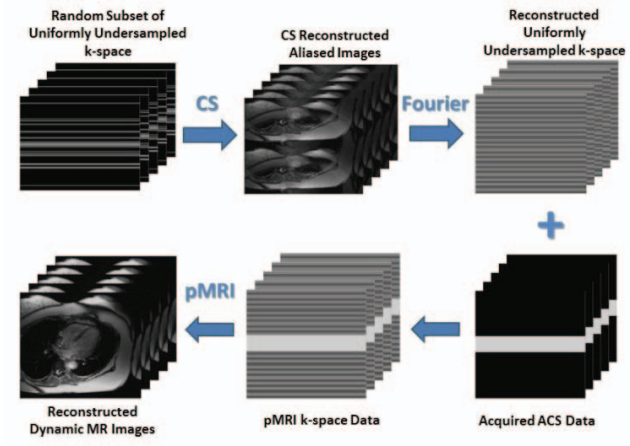



**Fig. 1.** Reconstruction procedure of proposed *k-t* CSPI method

is used here. In most CS-based dynamic imaging methods [18-22], the dynamic image series is assumed to be sparse in the spatial temporal frequency (*x-f*) domain. We use $\boldsymbol{\rho}_{\mathrm{aliased}}$ to represent the aliased image series to be reconstructed in *x-f* domain. The data consistency term is given by

$$\mathbf{d}_{u,r} = \mathbf{F}\boldsymbol{\rho}_{\mathrm{aliased}} \qquad (2)$$

where $\mathbf{F}$ is the Fourier transform in both *x* and *f* directions. In addition to the sparseness in *x-f*, we also assume the image series is also sparse in spatial and temporal total variations ( $\mathbf{TV}_{S-T}$ ). Specifically, the finite difference is calculated in 2-D spatial and 1-D temporal directions separately. The reconstruction of the aliased image series can be represented as

$$\min_{\boldsymbol{\rho}_{\mathrm{aliased}}} \left\| \boldsymbol{\rho}_{\mathrm{aliased}} \right\|_1 + \alpha \mathbf{TV}_{S-T}\left(\mathbf{F}_t \boldsymbol{\rho}_{\mathrm{aliased}}\right) \text{ s.t. } \left\| \mathbf{d}_{u,r} - \mathbf{F}\boldsymbol{\rho}_{\mathrm{aliased}} \right\|_2 \le \varepsilon. \qquad (3)$$

Where $\mathbf{F}_t$ represent Fourier transform along the temporal frequency direction. Non-linear conjugate gradient is used to solve the optimization problem. The reconstructed $\boldsymbol{\rho}_{\mathrm{aliased}}$ is then Fourier transformed in both spatial and temporal frequency directions to obtain the uniformly undersampled data in *k-t* space. The reconstructed data are then normalized based on the overlapping locations between the reconstructed and acquired *k*-space data.

#### 2.2.2 Step Two: Parallel Reconstruction Using Non-Linear GRAPPA

Our goal in this step is to further reconstruct the missing *k*-space data without the need for coil sensitivities. Nonlinear GRAPPA [23] is used here because the method has been shown to be less sensitive to the noise/error in the undersampled data than GRAPPA. Since the ACS date used for calibration is acquired directly, but the undersampled *k*-space data is CS reconstructed, there might be mismatch

between the two types of data. Normalization is performed to correct any mismatch. Specifically, we assume the reconstructed and acquired data differ by a scaling complex constant:

$$\mathbf{d}_{\text{recon}} = \beta \mathbf{d}_{\text{acq}} \ , \tag{4}$$

where $\beta$ is the scaling factor. The scaling factor is obtained for each frame using the *k*-space data at overlapping locations between the reconstructed and acquired data.

After normalization, nonlinear GRAPPA is performed. The core idea behind nonlinear GRAPPA is to use a nonlinear combination of the acquired data to represent the missing data. A truncated second-order polynomial has been shown to be efficient and effective and is thereby used here. The unacquired *k*-space signal $S_j$ is obtained by

$$\begin{aligned} S_j\left(k_y + r\Delta k_y, k_x\right) = {} & w_{j,r}^{(0)} \times 1 \\ & + \sum_{l=1}^{L} \sum_{b=B_1}^{B_2} \sum_{h=H_1}^{H_2} w_{j,r}^{(1)}(l,b,h) \times S_l\left(k_y + bR\Delta k_y, k_x + h\Delta k_x\right) \\ & + \sum_{l=1}^{L} \sum_{b=B_1}^{B_2} \sum_{h=H_1}^{H_2} w_{j,r}^{(2,0)}(l,b,h) \times S_l^2\left(k_y + bR\Delta k_y, k_x + h\Delta k_x\right) \\ & + \sum_{l=1}^{L} \sum_{b=B_1}^{B_2} \sum_{h=H_1}^{H_2-1} w_{j,r}^{(2,1)}(l,b,h) \times S_l\left(k_y + bR\Delta k_y, k_x + h\Delta k_x\right) \times \\ & \quad S_l\left(k_y + bR\Delta k_y, k_x + (h+1)\Delta k_x\right) \\ & + \sum_{l=1}^{L} \sum_{b=B_1}^{B_2} \sum_{h=H_1}^{H_2-2} w_{j,r}^{(2,2)}(l,b,h) \times S_l\left(k_y + bR\Delta k_y, k_x + h\Delta k_x\right) \times \\ & \quad S_l\left(k_y + bR\Delta k_y, k_x + (h+2)\Delta k_x\right), \end{aligned} \tag{5}$$

where $S_l$ is the acquired *k*-space data, $w$ denotes the coefficient set, $R$ represents the outer reduction factor, $j$ is the target coil, $l$ counts all coils, $b$ and $h$ transverse the acquired neighboring *k*-space data in $k_y$ and $k_x$ directions respectively, and the variables $k_x$ and $k_y$ represent the coordinates along the frequency- and phase-encoding directions, respectively. Once the missing *k*-space data is reconstructed for all coils, the final image is obtained by combining images from all coils using root of sum-of-squares (SoS).

## 3. EXPERIMENTS AND RESULTS

The feasibility of the proposed method was validated on a set of cardiac cine data acquired using a 2D true FISP sequence with a flip angle of 50 degree and TE/TR=1.87/29.9 msec. The full dataset was of 256×216×20×4 (#frequency encoding × #phase encoding × #frame × #coil). The FOV in the readout direction was 34 cm, bandwidth was 930 Hz/pixel, and the slice thickness was 6mm. For the proposed method, the reduction factor is $R_1$=2 for NL-GRAPPA and $R_2$ = 4 for CS, and ACS is 32 (net reduction factor of 4.23). Reconstruction using *k-t* FOCUSS with sensitivity encoding [8] was also performed. Variable density random sampling was applied to directly undersample the full *k*-space with the same net reduction factor. Because reconstruction is sensitive to the random sampling pattern, the best pattern among 12 trials was chosen for each method. All the computations were carried out on a Hewlett-Packard workstation with Xeon 3.40GHz CPU and 32GB RAM running MATLAB 2009a (Mathworks, Natick, MA). The computation time is 478s for the proposed method and 142s for *k-t* FOCUSS with SENSE. The image series that is Fourier reconstructed from the full *k-t* data was used as the reference for comparison. All images were zero-padded to display images with the right ratio representing the original FOV, and were normalized and shown on the same scale. Figure 2 compares a single frame of the dynamic reconstructions using full data, *k-t* FOCUSS with SENSE, and the proposed *k-t* CSPI. A zoomed region of interest (ROI) is also shown to reveal more details. The ROI is chosen at the area that has the most significant motion. The proposed method is seen to have fewer artifacts and preserve more details in the ROI than *k-t* FOCUSS with SENSE. The superiority of the proposed method is also demonstrated in the difference images.

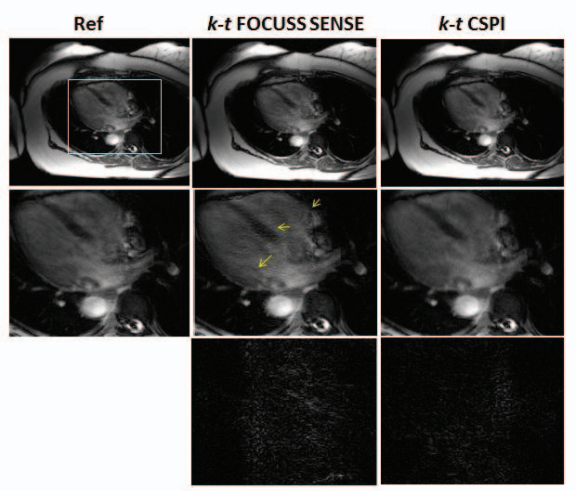



**Fig. 2.** Comparison among *k-t* FOCUSS with SENSE, proposed *k-t* CSPI, and reference. From top to bottom are: the full field of view, the zoomed ROI, and the corresponding difference image in a single frame of reconstruction.

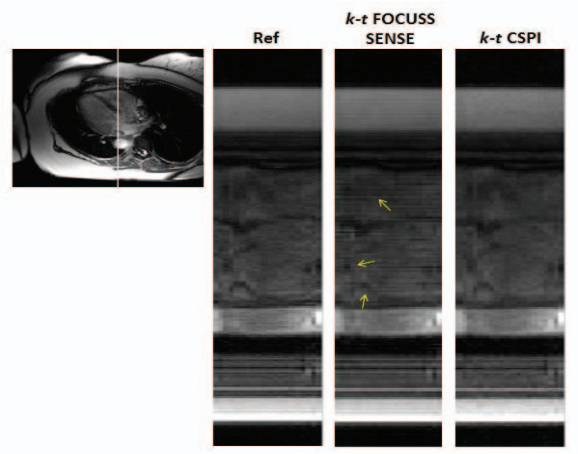



**Fig. 3.** Comparison of the temporal profiles. From left to right: white line indicating the location, reference, *k-t* FOCUSS with SENSE, and *k-t* CSPI.

We also show the temporal profile of the original, *k-t* FOCUSS with SENSE, and the proposed *k-t* CSPI in Fig. 3. The proposed method is able to better capture the temporal variations than *k-t* FOCUSS with SENSE.

To quantify the improvement of proposed method over *k-t* FOCUSS with SENSE, the frame by frame normalized mean-squared error (NMSE) between the reconstructed and reference ROIs were calculated and plotted in Fig. 3. Clearly, the proposed method outperforms *k-t* FOCUSS with SENSE at all frames in terms of NMSE.

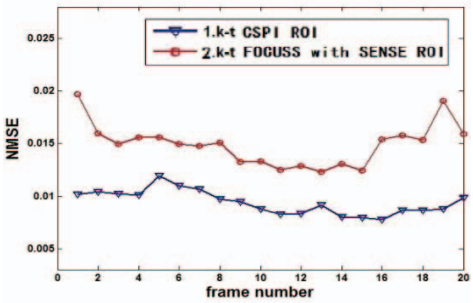



**Fig. 4**. Frame-by-frame plots of the NMSE for *k-t* FOCUSS with SENSE and *k-t* CSPI.

## 4. CONCLUSION

In this paper, a new method for dynamic MRI, named *k-t* CSPI, is developed. The method combines CS and nonlinear GRAPPA sequentially to improve the dynamic MRI reconstruction when the data acquisition is highly accelerated. The cardiac cine imaging experiment shows that the proposed method is able to preserve spatial details and temporal variations of dynamic images than the state-of-the-art dynamic imaging method that combines CS with parallel imaging.

## 5. ACKNOWLEDGMENT

This work was supported in part by the National Science Foundation CBET-0731226.

## 6. REFERENCES

[1] Z.-P. Liang, and P. C. Lauterbur, "An efficient method for dynamic magnetic resonance imaging", *IEEE Trans. Med. Imag.*, vol. 13, no. 4, pp. 677–686, Dec. 1994.

[2] J. J. van Vaals, M.E. Brummer, W. T. Dixon, H. H. Tuithof, H. Engels, R. C. Nelson, B.M. Gerety, J. L. Chezmar, and J. A. den Boer, "Keyhole method for accelerating imaging of contrast agent uptake," *J. Magn. Reson. Imag.*, vol. 3, no. 4, pp. 671–675, Aug. 1993.

[3] S. J. Riederer, T. Tasciyan, F. Farzaneh, J. N. Lee, R. C. Wright, and R. J. Herfkens, "MR fluoroscopy: technical feasibility," *Magn. Reson. Med.*, vol. 8, no. 1, pp. 1–15, Sep. 1988.

[4] B. Madore, G. H. Glover, and N. J. Pelc, "Unalising by Fourier-encoding the overlaps using the temporal dimension (UNFOLD), applied to cardiac imaging and fMRI," *Magn. Reson. Med.*, vol. 42, no.5, pp. 813–828, Nov. 1999.

[5] Z.-P. Liang, "Spatiotemporal imaging with partially separable functions," in *Proc. IEEE ISBI*, Washington DC, 2007, pp. 988–991.

[6] C. Brinegar, H. Zhang, Y.-J. L. Wu, L. M. Foley, T. K. Hitchens, Q. Ye, D. Pocci, F. Lam, C. Ho, and Z.-P. Liang, "Real-time cardiac MRI using prior spatial-spectral information," in *Proc. 31st Int. Conf. IEEE EMBS*, Minneapolis, MN, 2009, pp. 4383–4386.

[7] U. Sümbül, J. M. Santos, and J. M. Pauly, "Improved time series reconstruction for dynamic MRI," *IEEE Trans. Med. Imag.*, vol. 28, no. 7, pp. 1093–1104, Jul. 2009.

[8] N. Aggarwal, and Y. Bresler, "Patient-adapted reconstruction and acquisition dynamic imaging method (PARADIGM) for MRI," *Inverse Problems*, vol. 24, no. 4, pp. 045015-1 - 045015-29, Aug. 2008.

[9] J. Tsao, P. Boesiger, and K. P. Pruessmann, "*k-t* BLAST and *k-t* SENSE: dynamic MRI with high frame rate exploiting spatiotemporal correlations," *Magn. Reson. Med.*, vol. 50, no. 5, pp. 1031–1042, Nov. 2003.

[10] K.P. Pruessmann, M. Weiger, M.B. Scheidegger, and P. Boesiger, "SENSE: Sensitivity encoding for fast MRI," *Magn. Reson. Med.*, vol. 42, pp. 952-962, Nov. 1999.

[11] M.A. Griswold, P.M. Jakob, R.M. Heidemann, N. Mathias, V. Jellus, J. Wang, B. Kiefer, and A. Haase, "Generalized autocalibrating partially parallel acquisitions (GRAPPA)," *Magn. Reson. Med.*, vol. 47, pp. 1202-1210, Jun. 2002.

[12] E. J. Candès, J. Romberg, and T. Tao, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information," *IEEE Trans. Inf. Theory*, vol. 52, no. 2, pp. 489–509, Feb. 2006.

[13] D. Donoho, "Compressed sensing," *IEEE Trans. Inf. Theory*, vol. 52, pp. 1289 - 1306, Apr. 2006.

[14] M. Lustig, D. Donoho, and J. M. Pauly, "Sparse MRI: The application of compressed sensing for rapid MR imaging," *Magn. Reson. Med.*, vol. 58, no. 6, pp. 1182-1195, Dec. 2007.

[15] B. Liu. F. Sebert, Y. M. Zou, and L. Ying, "SparseSENSE: application of compressed sensing in parallel MRI", In *Proc. Int. Soc. Mag. Reson. Med.*, Toronto, Canada, 2008, p. 3154.

[16] D. Liang, B. Liu, J. J. Wang, and L. Ying, "Accelerating SENSE using compressed sensing," *Magn. Reson. Med.*, vol. 62, no. 6, pp. 1574-1584, Dec. 2009.

[17] M. Lustig and J. M. Pauly, "SPIRiT: Iterative self-consistent parallel imaging reconstruction from arbitrary *k*-space," *Magn. Reson. Med.*, vol. 64, pp. 457–71, 2010.

[18] M. Lustig, J. M. Santos, D. L. Donoho, and J. M. Pauly, "*k-t* SPARSE: high frame rate dynamic MRI exploiting spatio-temporal sparsity," in *Proc. Int. Soc. Mag. Reson. Med.*, Seattle, WA, 2006, p. 2420.

[19] T. Lang, and J. Ji, "Accelerating dynamic contrast-enhanced MRI using compressed sensing," in *Proc. Int. Soc. Mag. Reson. Med.*, Toronto, Canada, 2008, p. 1481.

[20] U. Gamper, P. Boesiger, and S. Kozerke, "Compressed sensing in dynamic MRI," *Magn. Reson. Med.*, vol. 59, no. 2, pp. 365–373, Feb. 2008.

[21] H. Jung, K. Sung, K.S. Nayak, E.Y. Kim, and J.C. Ye, "*k-t* FOCUSS: a general compressed sensing framework for high resolution dynamic MRI," *Magn. Reson. Med.*, vol. 61, no. 1, pp. 103–116, Jan. 2009.

[22] D. Liang, E. V. R. DiBella, R.-R. Chen, and L. Ying, "*k-t* ISD: dynamic cardiac MR imaging using compressed sensing with iterative support detection", *Proc. IEEE ISBI*, Chicago, IL, 2011, pp. 2064–2067.

[23] Y. Chang, D. Liang, and L. Ying, "A kernel approach to parallel MRI reconstruction", *Proc. IEEE ISBI*, Chicago, IL, 2011, pp. 389-392.