SIMULTANEOUS IMAGE RECONSTRUCTION AND SENSITIVITY ESTIMATION IN PARALLEL MRI USING BLIND COMPRESSED SENSING

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ABSTRACT

Parallel magnetic resonance imaging (MRI) reconstruction problem can be formulated as a multichannel sampling problem where solutions can be sought analytically. However, the channel functions given by the coil sensitivities in parallel imaging are not known exactly and the estimation error usually leads to artifacts or degraded SNR. In the context of parallel MRI, this work investigates the blind multichannel under-sampling problem where both the channel functions and signal are reconstructed simultaneously under sparseness constraints. We propose a novel algorithm to reconstruct both the coil sensitivities and image simultaneously from randomly undersampled, multichannel *k*-space data. The algorithm effectively applies the concept of compressed sensing (CS) to solve an underdetermined nonlinear problem, but is different from the conventional CS in that the sensing matrix is not known exactly. The proposed algorithm is shown to improve the reconstruction accuracy of SparseSENSE and L_1-SPIRiT when the same number of measurements is used.

Index Terms— parallel MRI, compressed sensing, blind deconvolution, Sparse BLIP, regularization

1. INTRODUCTION

Parallel imaging with phased array coils can be formulated as a multichannel sampling problem [1]. Based on Papoulis generalized sampling theorem, SENSE [2] is an optimal reconstruction method among many existing methods when exact knowledge of the coil sensitivities is provided. However, coil sensitivities are usually obtained from low resolution images or through a separate scan, where accuracy cannot be guaranteed. Inaccuracy in coil sensitivities may lead to noise amplification and artifacts at high acceleration factors. JSENSE [3] provides a framework to reconstruct both the image and coil sensitivities simultaneously. It models both coil sensitivities and the desired image as unknowns in the imaging equation, and the underlying nonlinear problem can be solved by alternating minimization [3] or Gauss Newton method [4].

With recent emergence of the compressed sensing (CS) theory [5,6] and its successful application in MRI [7], several techniques [8-10] have been developed to apply CS to SENSE. Among these techniques, Sparse SENSE [8] is a direct application of CS reconstruction on SENSE with random trajectories. It minimizes the L_1 norm of the image in the sparsifying transform domain under the data consistent constraint from the SENSE equation. Similar to SENSE, these methods still suffer from artifacts when coil sensitivities are inaccurate. On the other hand, L_1-SPIRiT [10] applies the sparseness constraint to the GRAPPA family [11], and thereby has the advantage of reconstruction without knowledge of coil sensitivities. However, it is computationally intensive, although effort has been made to reduce the complexity [12].

In this paper, we propose a new approach to parallel imaging reconstruction without the knowledge of coil sensitivities using CS. We name the method Sparse BLIP (application of CS for Blind Iterative Parallel imaging). Built on the formulation of JSENSE, the proposed method treats image and coil sensitivities both as unknowns in SENSE equation, but effectively incorporates the sparseness constraint to both the desired image and coil sensitivities. Subject to the data consistency constraint of JSENSE, the total variation of both image and sensitivities are minimized simultaneously. Although the underlying problem is non-convex, it can be decoupled into two convex optimization problems which can be solved in an alternating manner. The performance of the proposed method is compared with that of Sparse SENSE and L_1-SPIRiT.

2. BACKGROUND

2.1 SENSE and Sparse SENSE

In SENSE with arbitrary trajectories, the imaging equation can be written as

$$\mathbf{Ef} = \mathbf{d} \tag{1}$$

where $\mathbf{d}$ is the vector formed from all *k*-space data acquired from all channels, and $\mathbf{f}$ is the unknown image vector formed from the desired full field of view (FOV) image. The encoding matrix $\mathbf{E}$ consists of the product of Fourier encoding and modulation by the coil sensitivity over the image, i.e.

$$\mathbf{E}_{\{l,p\},q} = e^{-i2\pi \vec{k}_p \cdot \vec{r}_q} s_l(\vec{r}_q), \quad (2)$$

where p and q denote the indices for the k-space data and image pixels respectively, l is the coil index. To reconstruct the desired image $\mathbf{f}$ given knowledge of acquired data $\mathbf{d}$ and sensitivities $s_l(\vec{r})$, the least-squares method is usually used to solve Eq. (1) either directly or iteratively. Sparse SENSE [8] directly applies CS to SENSE by generalizing the formulation of Sparse MRI to sensitivity encoding. In data acquisition, the variable-density random undersampling scheme is used to sample the k-space for all channels. Specifically, the k-space data are randomly undersampled along the phase-encoding direction with denser sampling near the center of the k-space. In reconstruction, the problem is formulated as a constrained nonlinear convex program based on the SENSE equation (1):

$$\text{minimize } \|\mathbf{\Psi f}\|_1 \quad \text{s.t. } \|\mathbf{Ef} - \mathbf{d}\|_2 \le \varepsilon, \quad (3)$$

where $\mathbf{\Psi}$ is the sparse transformation matrix.

2.2. Summary of JSENSE

JSENSE [3] provides a framework to reconstruct the image and coil sensitivities simultaneously such that error in sensitivities is prevented from propagating to the final reconstruction. Specifically, instead of assuming that the sensitivity functions are known as in SENSE, JSENSE assumes that sensitivities are unknown variables to be solved for. Assuming a parametric model for the coil sensitivities with unknown parameters $\mathbf{a}$, the imaging equation in Eq. (1) becomes

$$\mathbf{E(a)f} = \mathbf{d}. \quad (4)$$

Then the unknown parameters $\mathbf{a}$ for coil sensitivities and the desired image $\mathbf{f}$ can be solved by finding a least- squares solution to Eq. (4):

$$\arg\min_{\{\mathbf{a},\mathbf{f}\}} \|\mathbf{d} - \mathbf{E(a)f}\|^2. \quad (5)$$

The above optimization problem does not guarantee a unique optimal solution in general. When the data is acquired at Nyquist rate, this problem is similar to the well-known multichannel blind deconvolution where the parametric model corresponds to finite impulse response filters for all channels. There are theoretical results and algorithms for exact reconstruction under certain conditions [13]. In parallel imaging, the conditions of uniqueness are not satisfied and the data are undersampled, which makes the problem much more complicated. The prior information on the smoothness of the coil sensitivity functions is exploited to regularize the otherwise highly underdetermined problem. In [3], a low-order polynomial model is assumed for the coil sensitivities. In [4], a pixel-based model is used but regularized by a Sobolev norm defined as $\|(I-\Delta)^{\lambda/2}\mathbf{a}\|$ to enforce smoothness on the sensitivities, where $\Delta = \partial_x^2 + \partial_y^2$ and λ is some chosen index. Nevertheless, the underlying optimization problem is non-convex in general and a good initial estimate of coil sensitivities from a set of low resolution images is needed for convergence to a solution that is close to the true one.

3. PROPOSED METHOD

3.1 Problem formulation

Motivated by the success of JSENSE, the proposed method also aims to reconstruct the desired image and coil sensitivities simultaneously. The data consistency term is thereby given by Eq. (4) where a pixel-based model is used for the coil sensitivities. The unknown parameter $\mathbf{a}$ represents all pixels of the sensitivities of all coils. The proposed method improves upon JSENSE by incorporating the concept of CS into the data consistency formulation. Specifically, the data is acquired with incoherent sampling and the reconstruction enforces sparseness constraints on both the image and sensitivity functions. For incoherence sampling, we use a random undersampling pattern in Cartesian grid with variable density as in [7]. The low frequency part of k-space is sampled more than the high frequency part. To resolve the ambiguity of non-unique solutions in reconstruction, the one minimizing the total-variation (TV) of both image and sensitivities in spatial domain is sought for as the optimal solution. Specifically, the image $\mathbf{f}$ and sensitivity functions $\mathbf{s}_l$ are jointly reconstructed through minimizing the energy function defined as

$$\mathbf{J}(\mathbf{f},\mathbf{s}_l) = \sum_{l=1}^{L} \left\| \mathbb{F}_D(\mathbf{s}_l \otimes \mathbf{f}) - \mathbf{d}_l \right\|^2 + \alpha \mathrm{TV}(\mathbf{f}) + \beta \sum_{l=1}^{L} \mathrm{TV}(\mathbf{s}_l), \quad (6)$$

where $\mathbb{F}_D$ is the 2-D Discrete Fourier operator with undersampling, $\otimes$ denotes the Hadamard product, i.e. pixel-wise multiplication of two matrix, L is the total coil number. The first data consistency term measures the fidelity to the data and comes from the acquisition model in Eq. (4). The last two regularization terms utilize the sparseness of the TVs of image and sensitivity functions in spatial domain where the TV operator is defined as:

$$\mathrm{TV}(\mathbf{f}) = \sum_{m=1}^{M}\sum_{n=1}^{N} \sqrt{\left(f_{m+1,n} - f_{m,n}\right)^2 + \left(f_{m,n+1} - f_{m,n}\right)^2}. \quad (7)$$

The positive constants α and β are adjusted to control the tradeoff between data consistency and the two sparseness priors. It is worth noting that our formulation is different from the one reported in [14] where CS was also used for JSENSE. A pixel-based model is used here for representation of coil sensitivities instead of a polynomial or Fourier model in [14]. Due to the high level of smoothness, the TV of the pixel-based coil sensitivity is usually sparser than the polynomial or Fourier coefficients.

Compared with the channel-by-channel reconstruction methods such as GRAPPA and L_1-SPIRiT, the proposed method is advantageous in that no root Sum-of-Squares (SOS) is needed. It not only saves computation when a large

number of channels is involved, but also avoids the assumption that SOS of the channels is spatially uniform. Because minimizing TV favors images with uniform intensity, the use of the TV constraint on image potentially generates a final reconstruction that is more uniform in intensity than that given by the SOS.

3.2 Implementation Using Alternating Minimization

When both **f** and $\mathbf{s}_l$ are unknowns, the joint optimization problem in Eq. (6) is beyond the conventional CS frame and becomes non-convex. It can be regarded as a blind CS problem where the sensing matrix is not given but is known to have certain specific structure. Under Eq. (6), the energy function **J** is still convex with respect to **f** if $\mathbf{s}_l$ is given, and similarly, **J** is also convex with respect to $\mathbf{s}_l$ if **f** is given. We therefore take a greedy approach to minimize the energy function by alternating between minimizing over f and minimizing over $\mathbf{s}_l$ as done in [3]. Specifically, the coil sensitivities are initially estimated using either a pre-scan or the calibration data fully sampled at the central *k*-space and then assumed to be given and plugged into Eq. (6). This step is equivalent to Sparse SENSE where TV is used to sparsify the image. We then use the reconstructed image from the previous step as a given **f** and find the updated coil sensitivities that minimize the energy function in Eq. (6). Subsequently, we alternate between updating the image and updating the coil sensitivities through iterations to obtain a sequence of minimizing $\left(\mathbf{f}^{(n)}, \mathbf{s}_i^{(n)}\right)$ until the energy function stops decreasing. We thereby reconstruct both the image and coil sensitivities at the end of the final iteration. Because the convergence to the global minimum is not guaranteed in this greedy approach, an accurate initial estimate of the unknown coil sensitivities is desired for high quality reconstructions. Nonlinear conjugate gradient (NLCG) algorithm with line search can be used to solve the above optimization problem.

4. EXPERIMENTAL RESULTS

The proposed Sparse BLIP method was evaluated on both phantom and *in vivo* brain datasets. SOS reconstructions from fully sampled data were shown as the reference for comparison. The state-of-the-art methods, Sparse SENSE [8] and L_1-SPIRiT [10], were also used to reconstruct from undersampled data and their results are shown to demonstrate the benefit of the proposed Sparse BLIP method. All reconstruction methods were implemented in MATLAB (MathWorks, Natick, MA) on a workstation (Hewlett-Packard, Palo Alto, CA) with a 2.66-GHz CPU and 3-GB RAM. The variable-density random undersampling pattern [7] was used with higher density toward the central *k*-space. The sensitivities for Sparse SENSE were estimated using the central *k*-space data. The initial sensitivities for Sparse BLIP were obtained from the randomly undersampled data. All reconstructed images for the same dataset were normalized and shown individually on the same scale for visual evaluations.

4.1 Phantom Experiment

The objective of the phantom experiment is to demonstrate the performance of the proposed method when the TV model for image and sensitivities is rather accurate. A phantom that is piecewise-constant was used whose TV is sparse. A T1-weighted scan was performed on the phantom using a two-dimensional spin echo sequence on a 3T commercial scanner (GE Healthcare, Waukesha, WI) with an eight-channel torso coil (echo time/pulse repetition time = 11/300 ms, FOV = 18 × 18 cm, matrix = 256 × 256, slice thickness = 1.7 mm). A net reduction factor of 5.95 was used with 24 Nyquist-sampled lines at central *k*-space. Fig. 1 presents the images reconstructed from the reduced data using three different methods. The SOS image is also shown for comparison. The parameters of the proposed method are $\alpha = 5\times10^{-6}$, $\beta = 6\times10^{-4}$, and the algorithm converges in 5 iterations. It is seen that the proposed Sparse BLIP method is able to remove the aliasing artifacts in L_1-SPIRiT and Sparse SENSE reconstructions and the Sparse BLIP reconstruction is very close to the SOS image.

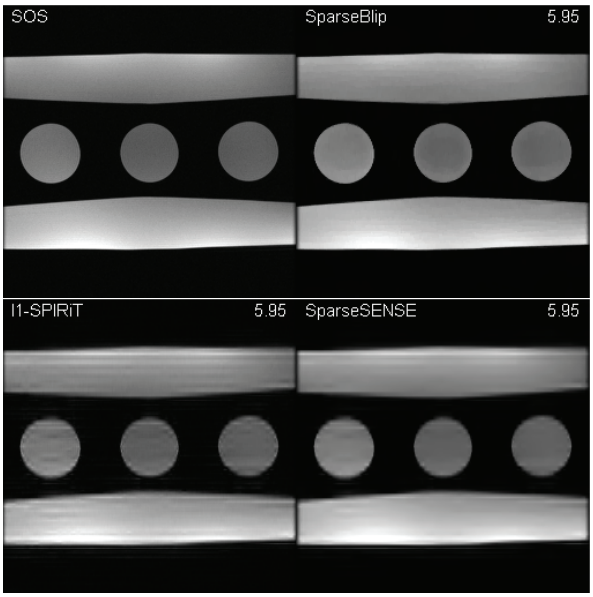


Figure 1. Comparison of phantom reconstruction results with 1-D undersampling at a reduction factor of 5.95.

4.2 *In vivo* Experiment

A 3-D MPRAGE dataset (TE/TR = 3.45/2530 ms, TI= 1100 ms, FOV = 25.6 cm, matrix = 256 × 256, slice thickness = 1.33 mm) acquired with an eight-channel head coil was obtained from http://www.nmr.mgh.harvard.edu/

~fhlin/codes/mprage_8ch_slice1.mat. Reconstructions in the axial direction are shown in Fig. 2. A 2D downsampling pattern with a net reduction factor of 16 was used with 32 × 32 Nyquist-sampled points at the central *k*-space. The parameters of the proposed method are $\alpha = 5\times10^{-6}$, $\beta = 6\times10^{-4}$, and the algorithm converges after 5 iterations. It is seen that the proposed method is able to better preserve the resolution than L_1-SPIRiT and SparseSENSE.

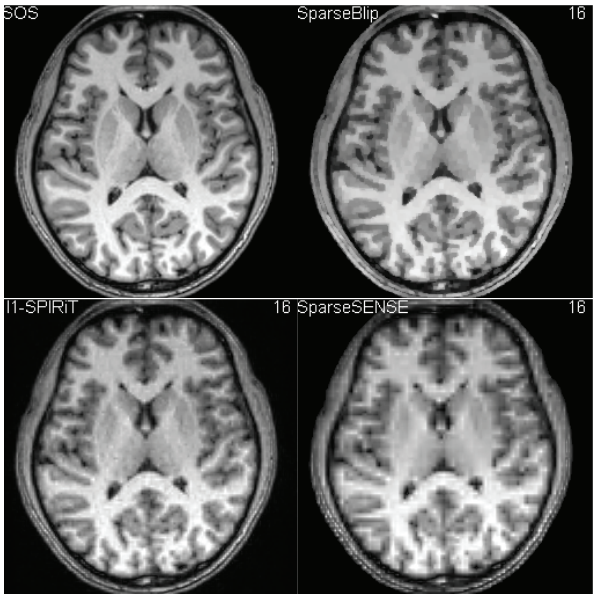


Figure 2. Axial reconstructions with 2-D undersampling.

5. CONCLUSION

We propose a novel method to reconstruct the image and coil sensitivities simultaneously from undersampled multichannel phased-array MRI data. Phantom and *in vivo* experimental results demonstrate that the proposed method can improve the quality of the reconstructed image over the existing L_1-SPIRiT and SparseSENSE methods.

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7. REFERENCES

[1] L. Ying, Z. P. Liang, "Parallel MRI using phased array coils: multichannel sampling theory meets spin physics," *IEEE Signal Processing Magazine*, vol. 27, pp. 90–98, 2010.

[2] K. P. Pruessmann, M. Weiger, M. B. Scheidegger, P. Boesiger, "SENSE: Sensitivity encoding for fast MRI," *Magn. Reson. Med.*, vol. 42, pp. 952–962, 1999.

[3] L. Ying, J. Sheng, "Joint image reconstruction and sensitivity estimation in SENSE (JSENSE)," *Magn. Reson. Med.*, vol. 57, pp. 1196–1202, 2007.

[4] M. Uecker, H. Hohage, K. T. Block, J. Frahm, "Image reconstruction by regularized nonlinear inversion- joint estimation of coil sensitivities and image content," *Magn. Reson. Med.*, vol. 60, pp. 674–682, 2008.

[5] E. J. Candès, J. Romberg, T. Tao, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information," *IEEE Trans. Inf. Theory*, vol. 52, pp. 489–509, 2006.

[6] D. Donoho, "Compressed sensing," *IEEE Trans. Inf. Theory*, vol. 52, pp. 1289–1306, 2006.

[7] M. Lustig, D. L. Donoho, J. M. Pauly, "Sparse MRI: The application of compressed sensing for rapid MR imaging," *Magn. Reson. Med.*, vol. 58, pp. 1182–1195, 2007.

[8] B. Liu, F. M. Sebert, Y. Zou, L. Ying, "SparseSENSE: randomly-sampled parallel imaging using compressed sensing," *Proceedings of the 16th Annual Meeting of ISMRM*, pp. 3154, 2008.

[9] D. Liang, B. Liu, J. Wang, L. Ying, "Accelerating SENSE using compressed sensing," *Magn. Reson. Med.* vol. 62, pp. 1574–1584, 2009.

[10] M. Lustig, J. M. Pauly, "SPIRiT: Iterative Self-consistent Parallel Imaging Reconstruction from Arbitrary *k*-Space," *Magn. Reson. Med.*, vol. 64, pp. 457–71, 2010.

[11] M. A. Griswold, P. M. Jakob, R. M. Heidemann, M. Nittka, V. Jellus, J. Wang, B. Kiefer, A. Haase, "Generalized autocalibrating partially parallel acquisitions (GRAPPA)," *Magn. Reson. Med.*, vol. 47, pp. 1202–1210, 2002.

[12] P. Lai, M. Lustig, A. C. Brau, S. Vasanawala, P. J. Beatty, M. Alley, "Efficient L_1-SPIRiT reconstruction (ESPIRiT) for highly accelerated 3D volumetric MRI with parallel imaging and compressed sensing," *Proceedings of the Annual Meeting of ISMRM*, pp. 345, 2010.

[13] G. Harikumar, Y. Bresler, "Blind restoration of images blurred by multiple filters: Theory and efficient algorithms," *IEEE Trans Image Processing*, vol. 8, pp. 202–219, 1999.

[14] C. Fernández-Granda, J. Sénégas, "L_1-norm regularization of coil sensitivities in non-linear parallel imaging reconstruction," *Proceedings of the Annual Meeting of ISMRM*, pp. 379, 2009.