HIGH EFFICIENT SURGICAL SIMULATION BASED ON STRUCTURED DICTIONARY

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ABSTRACT

This paper presents a method of real time 3D surgical simulation for deformable organs. It employs the technique of structured surface representation to decrease the description dimensionality, which allows for efficient deformation simulation by reducing the computational cost involved in the regular ODE solver. Majority of the deformation is estimated by surface reconstruction in the best-fit subspace. Experimental results are presented to demonstrate the feasibility of the method. The achieved accuracy is $2.3\% \pm 1.3\%$ in terms of normalized Euclidean distance and 0.71 ± 0.30mm for Hausdorff distance. The maximum error is below 2mm. Therefore, the proposed method achieves accuracy close to that of the traditional simulation method.

Index Terms— Surgical simulation, deformable organs, spherical harmonics, structured dictionary

1. INTRODUCTION

Real time surgical simulation [1] has become an important tool in the field of surgery for operation planning, training of surgeons in the image-guided surgery or minimally invasive surgery. The simulation tool is attractive because it avoids the use of patients for skill practice and ensures that trainees have had some practice before treating humans.

The techniques developed for surgical simulation have evolved quickly. Most of of those methods fall into the following three categories. The first type is heuristic methods, including deformable splines [2], spring-mass models [3] and linked volume [4]. These methods usually integrate elastic properties and are simpler than alternatives like Finite Element Methods (FEM) at the cost of limited realism. The second category is based on the laws of continuum mechanics, including FEM [5] and boundary element methods (BEM) [6]. In practice, the ideal mechanical models have to be simplified significantly in order to obtain real-time performance. The third hybrid methods aim to combine the merits of the above two approaches. Based on the expected kind of interaction, the hybrid methods divide a deformable object into different sections, each of which is modeled with an appropriate model to increase the overall simulation efficiency.

Each of the aforementioned methods has its strengths and weaknesses in terms of computational cost, simulation realism and general applicability under various conditions. The currently widely used spring-mass model is superior to others from the perspective of simulation speed due to its simple mesh structure. Further, the mesh simplicity also allows for any topological manipulations like incisions. However, the drawbacks of spring-mass model are the slow global propagation of deformations and predisposition to oscillate during the iteration process. The continuum mechanic methods achieve more realism by sacrificing the computational speed. Although fast finite element (FFE) method and tensor precomputation method alleviate the computational burden by pre-computing the inversion of the stiffness matrix, they are limited to models with constant topologies so they don't accommodate the common procedures such as cutting or stitching. Regular FEM and BEM considering dynamic mesh structures are usually hampered by the large amount of computing capacity for massive matrix inversion, which prevents the implementation of real time simulation. Recently, model reduction techniques [7] are proposed to achieve real time simulation by reducing the degree of freedom of the model, but they suffer from limited generalizability of the model.

Realizing the current challenges, this paper proposes a new approach for real time surgical simulation by bringing in the merits of structured dictionary. It relies on our observation that a deformable surface actually can be represented in one of the low dimensional subspaces embedded in a structured dictionary [8]. The main contributions of this paper include: (1) it presents an algorithm for efficient deformable organ simulation by integrating structured surface representation to allow for sparse surface point evaluations; (2) it provides the experimental results to demonstrate the feasibility of the proposed simulation method, achieving precision comparable to that of the traditional models.

2. ALGORITHM

The proposed algorithm applies the technique of structured surface representation into the traditional surface modeling approaches like mass-spring model. The goal is to reduce the number of surface vertices whose displacements in each iteration need to be updated by solving the ordinary differential equation (ODE). The majority of vertices' displacements are calculated by surface reconstruction in a best-fit subspace with efficiency higher than ODE method.

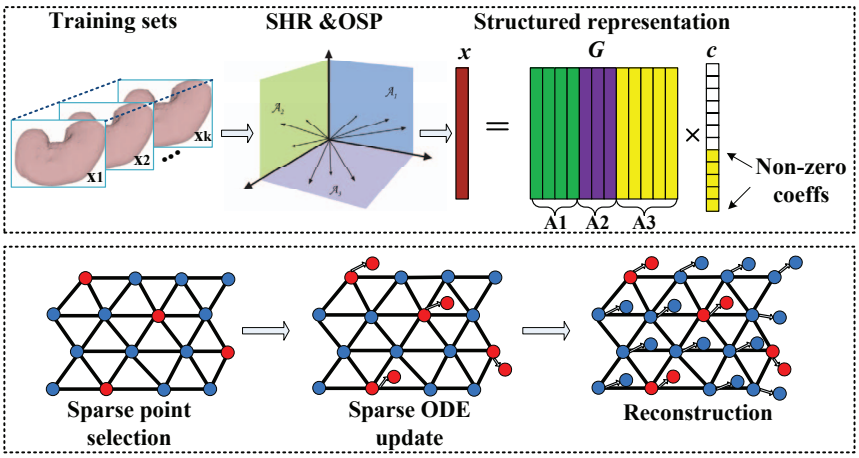


Fig. 1. Framework of real time 3D surgical simulation based on structured dictionary.

Fig. 1 outlines the framework of the method. The flowchart in the upper rectangular illustrates the training procedures. Spherical harmonic representation (SHR) transforms the training data set into the harmonic domain where orthogonal subspace pursuit (OSP) method [8] is conducted to identify the subspaces. Consequently, each surface x can be represented in the structured dictionary $\mathbf{G}$ with a block sparse vector c. The bottom part of Fig. 1 illustrates the real time simulation process. According to the trained subspaces, the sparse points are selected (red nodes), and displacements at those points are updated by the ODE solver. Deformations at other points (blue nodes) are reconstructed based on a surface reconstruction algorithm.

2.1. Training

The main steps involved in the training stage are: (1) SHR to decrease the length of surface descriptors and filter out the high frequency noise for achieving better homology among the training surfaces; (2) OSP to identify the subspaces that can generalize the deformation to be simulated. The training data set can be obtained by running the surgical simulator with specific setups.

Suppose that there are K training surfaces, and each surface $\vec{x}_k$ $(1 \leq k \leq K)$ contains N vertices. According to [8], each $\vec{x}_k$ can be expanded as

$$\vec{x}_k = \mathbf{Y}\vec{f}_k \tag{1}$$

where $\mathbf{Y}$ of size $N \times (L+1)^2$ denotes the SHR basis matrix with L as the maximum harmonic degree, and $\vec{f}_k$ is the coefficient vector. Then K training frames can be characterized by $F = [\vec{f}_1\ \vec{f}_2\ ...\ \vec{f}_K]$ in the transformed harmonic domain.

OSP method is then applied to the set of coefficient vectors $\vec{f}_k$. If there are totally J subspaces identified from $\mathbf{F}$, a structured dictionary constructed by concatenating all deformation subspaces is established as $\mathbf{D} = \bigcup_{i=1}^{J}\{\mathbf{A}_i\}$, with dimension $I = \sum_{i=1}^{J} n_i$. Since each vector $\vec{f}_k$ lies in one of the subspaces, $\vec{f}_k$ can be represented in the structured dictionary with a block sparse vector $\tilde{\vec{c}}_k$. Consequently, $\mathbf{F}$ can be factorized as

$$\mathbf{F} = \mathbf{DC} \tag{2}$$

where $\mathbf{C} = \left| \begin{array}{cccc} \tilde{\vec{c}}_1 & \tilde{\vec{c}}_2 & \cdots & \tilde{\vec{c}}_K \end{array} \right|_{I \times K}$ is the corresponding coefficient matrix with block sparsity.

Integrating the above two steps, each deformation $\vec{x}_k$ can be sparsely represented in the original spatial domain as:

$$\vec{x}_k = \mathbf{YD}\tilde{\vec{c}}_k = \mathbf{G}\tilde{\vec{c}}_k = \mathbf{G}_i\vec{c}_k$$

where $\mathbf{G} = \mathbf{YD}$ with size of $N \times I$ is the desired structured dictionary in the spatial domain and $\vec{c}_k$ is the non-zero coefficients of $\tilde{\vec{c}}_k$. $\mathbf{G}$ is inherently structured by subspaces of $\mathbf{G} = \bigcup_{i=1}^{J}\{\mathbf{G}_i\}$ with $\mathbf{G}_i = \mathbf{YA}_i$ of size $N \times n_i$.

2.2. Real time simulation

Sparse Position Selection

The first step during the real time simulation stage is to determine the surface point locations where displacements are updated with the ODE solver. This selection is only performed once before the real time simulation. The principle is to find a well conditioned sub-matrix $\widetilde{\mathbf{G}}_i$, such that the coefficient vector $\vec{c}$ can be stably estimated with the least square (LS) estimator from the following linear system

$$\widetilde{\vec{x}} = \widetilde{\mathbf{G}}_i\vec{c} \tag{3}$$

where $\widetilde{\vec{x}}$ is the m sparse surface samples (m must be larger than maximum n_i) and $\widetilde{\mathbf{G}}_i$ stands for the sub-matrix containing corresponding m rows of $\mathbf{G}_i$. $S = \{s_1, s_2, ..., s_m\}$ denotes the m indices of the selected points, highlighted as red nodes in Fig. 1. Those points are selected with the "random walk" method proposed in our previous work [8]. Notice that

the surface index k is neglected in Eq. 3 for illustration convenience. So, the deformation $\vec{x}$ can be reconstructed completely in the proper subspace with estimated $\hat{\vec{c}}$.

Sparse ODE Update

The second step is to compute the displacements at those selected positions S. Without loss of generalization, the spring-mass model is applied in this paper to illustrate how the efficiency of the simulation is improved using the structured dictionary. Each point (node) x^j $(1 \leq j \leq N)$ on surface $\vec{x}$ is connected to the direct neighbors with springs. According to the Newton's law of motion, the force equilibrium for each point j of the mesh can be established as

$$m^j \frac{d^2 x^j}{dt^2} + \gamma \frac{dx^j}{dt} + f_{int}^j = f_{ext}^j \tag{4}$$

where m^j is the mass of each node, x^j is the current coordinate, γ is the viscous friction coefficient, f_{ext}^j is the external force acting on node j, and f_{int}^j is the internal force trying to combat the external force and preserve the initial position. f_{int}^j is defined as

$$f_{int}^j = \Sigma_{n \in \Gamma_j} k_n^j \frac{|x^n - x^j| - |l_n^j|^0}{|x^n - x^j|}(x^n - x^j) \tag{5}$$

where Γ_j is the indices of neighbors of x^j, k_n^j evaluates the rigidity of the spring between the nodes n and j, and $|l_n^j|^0$ is the natural length of the spring.

With the traditional spring-mass simulation method, the set of ODE for all nodes is solved by numerical approaches such as Euler's method. In our proposed method, ODE update at each iteration is involved only at the selected nodes indexed by S. Deformation at other nodes are estimated by reconstructing the complete deformation in the best-fit subspace, as detailed in the next step.

Complete Deformation Reconstruction

The updated coordinates at the selected sparse nodes are the input to the complete deformation reconstruction. Considering the time factor during the simulation course, the linear system in Eq. 3 is rewritten as

$$\widetilde{\vec{x}}^{\,t+1} = \widetilde{\mathbf{G}}_i \vec{c}^{\,t+1} \tag{6}$$

Since the selection of sparse position S ensures the above linear system is well-conditioned, the equation system can be solved by applying the LS constraint. The remaining question is to determine the best-fit subspace for reconstruction. This is achieved by evaluating the reconstruction error at positions indexed by S in each subspace and choosing the subspace with the least reconstruction error as the best-fit subspace. Mathematically, the best-fit subspace i^* is selected by

$$i^* = \min_{1 \leq i \leq J} (\|\widetilde{\vec{x}}^{\,t+1} - \widetilde{\mathbf{G}}_i \hat{\vec{c}}^{\,t+1}\|_2) \tag{7}$$

Finally, the complete deformation is reconstructed as

$$\widehat{\vec{x}}^{\,t+1} = \mathbf{G}_{i^*} \widehat{\vec{c}}^{\,t+1} \tag{8}$$

To summarize the proposed algorithm, the sequential steps are further outlined as follows.

- *Step0:* Initialization: $t = 0$, $\vec{x} = \vec{x_0}$, set f_{ext}^j at designated nodes with initial values;
- *Step1:* t=t+1, compute $\widetilde{\vec{x}}^{\,t+1}$ at selected position S according to previous $\vec{x}^{\,t}$ with ODE solver;
- *Step2:* Reconstruct $\widehat{\vec{x}}^{\,t+1}$ with the structured dictionary;
- *Step3:* Replace $\widehat{\vec{x}}^{\,t+1}$ at locations indexed by S with known values $\widetilde{\vec{x}}^{\,t+1}$;
- *Step4:* Go back to Step1 until iteration is terminated.

3. EVALUATION

To verify the feasibility of the proposed approach, we implemented the algorithm in MATLAB. Initially, a porcine kidney model is segmented from a set of 3D magnetic resonance images with isotropic spatial resolution of 1.2 mm. The maximum spherical harmonic level L is set as 30, and number of vertices N is 4002. The training data is generated by driving the traditional model with initial forces exerted at certain points. The testing data is collected by stimulating our model with initial conditions at different nodes with different force levels. The simulation accuracy is evaluated in terms of Hausdorff distance and $EOF = ||\hat{\vec{x}} - \vec{x}||_2 / ||\vec{x}||_2$ as the normalized Euclidean distance between the estimated surface relative to the ground truth. The simulated deformations generated by the traditional spring-mass model with the same initial condition and parameters are used as the ground truth for comparison.

Subspace Generalization

Since the generalization of the identified subspaces indicated by the representation precision of the surfaces to be simulated is critical for high accuracy simulation, the subspace projection error is first evaluated. There are five subspaces on X and

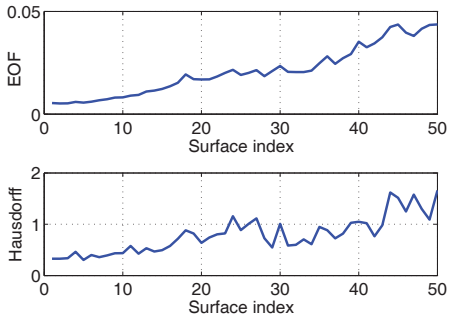


Fig. 2. Projection error in terms of EOF and Huasdorff distance.

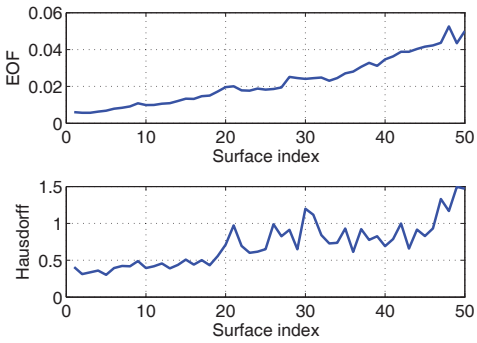


Fig. 3. Simulation error in terms of EOF and Huasdorff distance.



Fig. 4. Example of ODE generated deformation (left) and corresponding simulated deformation using the proposed method.

Y axes respectively, and three subspaces on Z axis, with maximum subspace size of 100. Note that, the low dimensionality of the trained subspace enables fast deformation simulation. The upper subplot in Fig. 2 shows the projection error in terms of EOF, and the bottom plot corresponds to the Hausdorff distance. Specifically, the average EOF error is $0.82\% \pm 0.39\%$ and the Hausdorff distance is 0.25 ± 0.11mm. This low error rate demonstrates the good generalization of the subspaces in the structured dictionary to the deformations to be simulated.

Simulation Accuracy

The simulated deformations generated using the proposed method is compared with the corresponding surfaces simulated with the traditional spring-mass model. Similarly, the upper subplot in Fig. 3 shows the simulation error in terms of EOF, and the bottom plot demonstrates the Hausdorff distance. The X axis corresponds to the surface indices as time evolves. The average EOF error is $2.3\% \pm 1.3\%$ with a maximum error of 5.3% and the Hausdorff distance is 0.71 ± 0.30mm with a maximum error of 1.50mm. Fig. 4 provides an example of the deformation generated by the traditional ODE model (left) and the corresponding simulated deformation using the proposed method (right). We can see that the two surfaces are very similar. These experiment results demonstrate that the accuracy of the proposed simulation algorithm is close to that of the mass-spring model.

One may notice that both the projection error in Fig. 2 and simulation error in Fig. 3 increase gradually as simulation progresses. The two main reasons for this issue are: (1)subspaces' decreased generalizability to later deformations, and (2) error propagation from previous estimation to following ones. The possible solutions are to increase the generalizability of the dictionary by training on more representative dataset and to increase the numerical stability by forming a better conditioned linear system.

4. CONCLUSION

This paper proposed an algorithm of real time 3D surgical simulation of deformable organs. It applies the technique of structured surface representation to achieve high simulation efficiency by reducing the computational cost involved in the regular ODE solver. Majority of the deformation is estimated by surface reconstruction in the best-fit subspace. The experimental results demonstrated the feasibility of the method. It achieves an accuracy of less than 2mm in terms of Hausdorff distance. The remaining challenge of the proposed method is the error propagation issue. As shown in Fig. 4, error increases as simulation takes place, so proper solution is desired to periodically correct the simulated deformation.

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