RECONSTRUCTION OF BONE MICROSTRUCTURE FROM FEW PROJECTIONS WITH CONVEX-CONCAVE AND NON LOCAL REGULARIZATION

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ABSTRACT

This work investigates a new method based on discrete tomography for the reconstruction of binary cross-sections of bone micro-structure from a small number of projections. While priors based on Markov random fields have been previously considered for the reconstruction of binary images, we propose to improve the regularization term by introducing long range smoothness constraints. To this aim, we propose to use a convex-concave deterministic optimization approach coupled with a non local regularization. Applications to 256×256 bone cross-section images provides good results from only 20 projections, even in the presence of additive gaussian noise.

Index terms – X-ray Imaging, Discrete tomography,Convex-concave regularization, Non local regularization

1. INTRODUCTION

The early diagnosis of osteoporosis, a frequent bone fragility disease in the elderly population, is important to prevent fractures and their complications. While the clinical diagnosis is based on dual X-ray absorptiometry, this technique is limited to predict the individual fracture risk. 3D bone microarchitecture, playing a significant role in the biomecanical properties of bone, is increasingly assessed in vitro by micro-CT techniques [1]. While High Resolution periphal QCT systems have recently appeared on the market to image bone microarchitecture in vivo at peripheral sites, the spatial resolution of such systems remains of the same order of magnitude as the mean trabecular size (100-150μm), challenging the quantitative analysis of the 3D microstructure [2]. A main limitation to improve spatial resolution in vivo is the resulting increase of radiation exposure. The reduction of radiation dose can be achieved by limiting the number of projections. However, the tomographic reconstruction problem becomes ill-posed and may be regularized with TV terms [3]. Discrete tomography methods assuming that the image only includes a finite number of discrete values have also been proposed [4]. For discrete tomography, various approaches have been investigated to solve the reconstruction problem by minimizing a functional that incorporates the projections and the constraints [4, 5]. These algorithms solve the optimization problem with stochastic techniques like simulated annealing [6], or convex analysis methods [7, 8, 9]. Algebraic iterations and linear programming relaxation approaches to the combinatorial reconstruction problem of discrete tomography have also been investigated [5]. In the field of discrete tomography, Markov Random Field modeling has been much used. In this approach, clique potentials are summed up over local pixel configurations [10]. Yet, these priors describe only short range correlations between the pixels.

In this work, we investigate a new method based on discrete tomography for the reconstruction of binary cross-sections of bone micro-structure [11]. In this application, the solution is made up of connected and homogeneous regions with tubular zones and the function to be reconstructed can be assumed to be binary-valued. The main objective of this paper is to improve the Markov random field priors with long range smoothness constraints. To this aim, we propose to use a convex-concave deterministic optimization approach coupled with a non local regularization. Recently, non local regularization techniques have attracted much attention [12, 13] but to our knowledge, these techniques have not been applied to discrete tomography so far.

This paper is organized as follows. In the first section, we state the optimization problem and we present the convex-concave optimization approach. In the second section, the non local regularization methodology used for our reconstruction problem is detailed. The numerical results are presented and discussed in the last section.

2. BACKGROUND ON DISCRETE TOMOGRAPHY WITH THE CONVEX-CONCAVE APPROACH

2.1. Objective functional

The discrete tomography problem is associated with an underdetermined linear system of equations with the projection matrix R:

$$Rf = p \quad f = (f_1, ...f_n) \in \{0,1\}^n \tag{1}$$

relating the pixel values $(f_i)_{1\leq i\leq n}$ of the image and the measured projection values p. Following [8, 9], the initial discrete tomography problem to be solved can be written as a functional minimization, $min_{f\in[0,1]^n}\{E(f,\alpha,\beta)\}$, with the functional $E(f,\alpha,\beta)$ given by:

$$E(f,\alpha,\beta)=\frac{\|Rf-p\|^2}{2}+\frac{\alpha}{2}<f,Lf>+\beta\frac{<f,(e-f)>}{2} \quad (2)$$

where e=(1,.....1) , α and β are regularization parameters and L is a smoothing matrix. The objective function combines the projection error term and a smoothness term that increases the spatial homogeneity. The last term is a concave functional that favors integer solutions. The optimization problem has been relaxed and the integer constraint $f_i\in\{0,1\}$ has been replaced by the constraints $f_i\in[0,1]$. If the constant terms are ignored, the objective function can be written as the difference of two convex functions:

$$E(f,\alpha,\beta)=\frac{<f,Qf>}{2}+<q,f>+\frac{\beta<f,(e-f)>}{2} \quad (3)$$

with $Q=R^tR+\alpha L$ and $q=-R^tp$

2.2. The convex-concave approach

A difference of convex functions program (or D.C program) consists to obtain a minimum of the form, $inf\{E(f)=h_1(f)-h_2(f),\ \ f\in\mathbb{R}^n\}$ where h_1 and h_2 belongs to Γ_0, the set of proper lower semi-continuous convex functions on $\mathbb{R}^n$. [8, 9]. This class of non convex optimization problem can be solved by the following algorithm: choose $f^0\in Dom(h_2)$ and for $k\geq 0$, compute

$$g^k\in\partial h_1(f^k),\ \ f^{k+1}\in\partial h_2^*(g^k) \quad (4)$$

where Dom(h_1) is the domain of the function h_1, $\partial h_1(f)$, the sub-differential of h_1 at f, and h_2^* the conjugate of h_2. Then, it can be shown that every limit point f^* of f_k is a critical point of the difference of the two convex functions, h_1 and h_2. Following Schule et al [8, 9], the functional $E(f,\alpha,\beta)$ was decomposed as $E(f,\alpha,\beta)=h_1(f,\alpha)-h_2(f,\beta)$ with:

$$h_1(f)=\frac{<f,\lambda If>}{2}+1_C(f) \quad (5)$$

$$h_2(f,\beta)=\frac{<f,[(\lambda+\beta)I-Q]f>}{2}-<q+\frac{\beta e}{2},f> \quad (6)$$

where $C=[0,1]^n$, and λ is an upper bound of the largest eigenvalue of Q. The update rules corresponding to (5-6) are given by:

$$g^k=[(\lambda+\beta)I-Q]f^k-(q+\beta e/2) \quad (7)$$

$$f_i^{k+1}=\begin{cases}0 & \text{if } g_i^k\leq 0\\ 1 & \text{if } g_i^k\geq\lambda\\ g_i^k/\lambda & \text{otherwise}\end{cases} \quad (8)$$

3. THE PROPOSED NON LOCAL REGULARIZATION

The former DC approach[8, 9] uses a standard smoothness prior based on the Laplacian:

$$<f,Lf>=\sum_{i=1}^{n}\sum_{j\in N(i)}(f_i-f_j)^2 \quad (9)$$

where N(i) denotes a 4-neighborhood of the pixel i.

In the following, we propose to extend this approach to a non local graph-based regularization. Recently, the use of non local interactions to take account of complex structures within images has been much investigated. [12, 13]. Yet, to our knowledge, these techniques have not been used in the context of discrete tomography.

A bone cross-section can be considered as a binary valued function $f: V\rightarrow\mathbb{R}$ defined on a weighted graph G=(V,E,ω) which consists of a set of vertices V, a set of edges $E\subset V\times V$ and a weight function $\omega: E\rightarrow\mathbb{R}$ defined on the edges. In this work, the vertices set is simply the cartesian grid. Following [13] the directional derivative of a function, f, at a vertex u, and over an edge (uv) is defined by $\partial_v f(u)=\sqrt{\omega(u,v)}(f(v)-f(u))$ and the gradient operator is the vector operator defined by $\nabla_\omega f(u)=(\partial_{v_1}f(u),.....,\partial_{v_1}f(u))$ where $v_1,....v_k$ belongs to the neighborhood V(u) of the vertex u. The regularization functional associated to the L_2 norm of the graph gradient is

$$R(f,w)=\frac{\sum_u\sum_{v\in V(u)}w(u,v)(f(u)-f(v))^2}{2} \quad (10)$$

The Laplace operator used for the minimization of this functional generalizes the classical Laplacian smoothing operator:

$$L_\omega Y(u)=\sum_{v\in V(u)}w(u,v)(f(u)-f(v)) \quad (11)$$

The operator L_ω is linear, positive and semidefinite In addition to the classical Laplacian operator, the following weight function has been used:

$$w(u,v)=\frac{1}{\|u-v\|^2}exp(-\frac{f(u)-f(v)}{2\sigma^2}) \quad (12)$$

for various values of σ. This type of smoothness prior generalizes the classical Markov random field modeling [9, 10].

4. RESULTS AND DISCUSSION

4.1. Simulations details

In the following, the projection matrix R is the one implemented in the Matlab Toolbox. In order to calculate an upper bound of the largest eigenvalue λ_{max} of the Q matrix, and for a given x_0, we define the sequences $\{x_n\}$ and $\{\rho_n\}$ by the

relations $x_{n+1} = Q^t Q x_n$ and $\rho_n = \frac{\|x_n\|}{\|x_{n-1}\|}$. It can be shown that $\lim_{n\to\infty} \rho_n = \|Q\|^2 = \lambda^2_{max}$. We have used a continuation method and an increasing sequence $\{\beta_k\}_{k\geq 0}$ of regularization parameters. When $\beta \to \infty$, the solution of the relaxed problem with $f_i \in [0,1]$ is converges towards a local optimum the solution of constrained problem $f_i \in \{0,1\}$ [8, 9]. The algorithm is thus initialized with $f_0 = e/2$ and $\beta_0 = 0$. For a given value of β_k, f_i is computed until convergence. After convergence of this inner loop, β is incremented with $\Delta\beta = 1$, the current solution is the new initial guess and the iterations (10-11) are repeated. The program was stopped when for all f_i, $max(f_i, 1 - f_i) < 10^{-3}$. The image is then projected on $\{0,1\}$ and the final reconstruction cross-section is a binary one.

4.2. Numerical results

The method was applied to simulated projections of an experimental bone cross-section acquired with synchrotron micro-CT (voxel size : 15 μm) [14]. Figure 1 illustrates the 256x256 bone cross-section reconstructed from 400 projections with 400 rays per projections with Filtered Back Projection (FBP). This reference image will be denoted f^*.

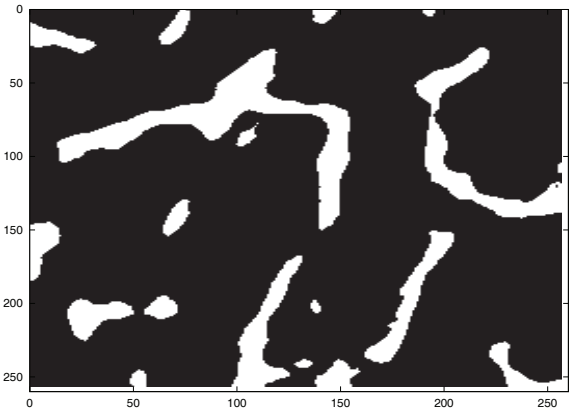


Fig. 1. Reconstruction of the bone cross section from 400 projections with the FBP algorithm

In the following, we consider the reconstruction from only 20 equally spaced projections with the proposed methods. The reconstructed images with the classical Laplacian operator and the proposed non local regularization are displayed in Figure 2 and 3 respectively. In this work, an upper bound of λ_{max} is 600 and the regularization parameter α, was chosen by trial and errors ($\alpha = 25$). Numerous isolated false detections are present with the Laplacian regularization with 9.5% of misclassified pixels. A better reconstruction is obtained with the non local regularization with 7.05% of misclassified pixels. This method provides large connected regions but some errors are still present on the borders of homogeneous regions. The evolution of the relative root mean square error $\|f^* - f_n\|_{l_2}/\|f^*\|$ has been studied as a function of the number of iterations. The results are displayed on Figure 4 for the classical Laplacian operator or the weight functions w defined in Equation (17). The best results were obtained by trial and errors with $\sigma = 1.5$. Better convergence results are obtained with the non local regularization functional. And therefore this approach proves to be very useful when long range correlations exist between the pixels leading to large connected homogeneous regions. As a first approach to study the stability of the reconstruction algorithm, simulations have been carried out in the presence of white noise on the projections for a low signal to noise ratio of 10 dB. The evolution of the relative root mean square error is displayed on Figure 4 for the Laplacian regularization (12% of misclassified pixels) and the non local regularization (9.5% of misclassified pixels). Due to the smoothing effect of the regularization terms, good reconstructions are obtained up to a relatively high noise level.

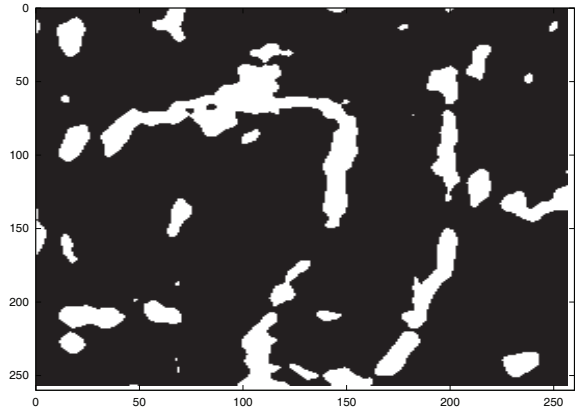


Fig. 2. Reconstruction of the bone cross section from 20 projections with the DC approach and classical Laplacian regularization

5. CONCLUSION

This work introduced convex-concave optimization methods in tomographic reconstruction of binary bone microarchitecture from a limited number of views. To improve further the performance of the algorithm obtained with classical smoothness priors, we proposed a non local regularization method with a smoothness prior favoring large homogeneous regions. The method was applied to simulated data showing a very small percentage of error from only 20 projections. Compared to the initial 400 projections with FBP, this represents a reduction of dose of a factor 20. The former approach is stable in the presence of noise and may be applied to larger images. Its main limitations are the binary assumption and the trial and error choice of the regularization parameters. Moreover, some errors are localized at the borders between

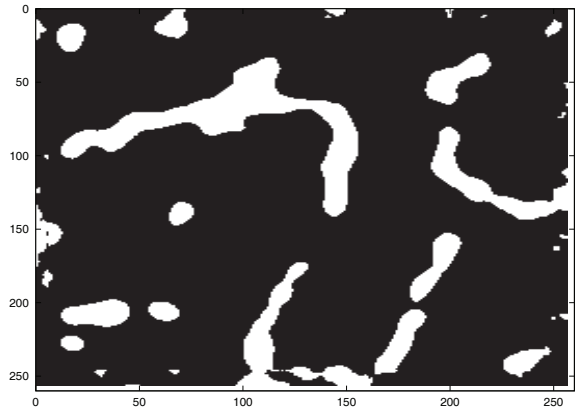


Fig. 3. Reconstruction of the bone cross section from 20 projections with the DC approach and the non local regularization

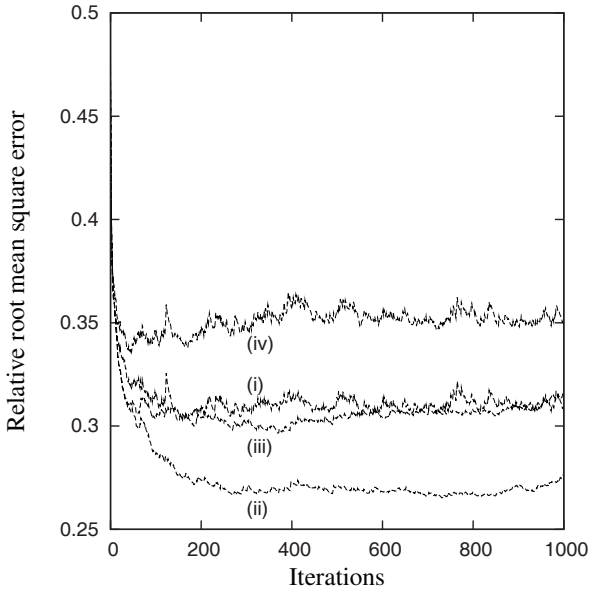


Fig. 4. Evolution of the relative root mean square error for (i) classical laplacian regularization (ii) non local regularization with the weight function ω (iii) non local regularization with the weight function ω and noisy projection data (SNR=10 dB) (iv) classical laplacian regularization and noisy projection data (SNR=10 dB).

different valued regions. A tradeoff is obtained between the smoothness of the bones and the missing of fine structures. The effect TV regularization or of additional stochastic change of the border regions will be investigated in future works on bone cross sections with various structural parameters.

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