SUPER RESOLUTION OF 3D MRI IMAGES USING A BIVARIATE LAPLACIAN MIXTURE MODEL CONSTRAINT

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ABSTRACT

In multi-slice magnetic resonance imaging (MRI) the resolution in the slice direction is usually reduced to allow faster acquisition times and to reduce the amount of noise in each 2-D slice. To address this issue a number of super resolution (SR) methods have been proposed to improve the resolution of 3D MRI volumes. These methods typically involve the use of prior models of the MRI data as regularization terms in an ill-conditioned inverse problem. However; an inappropriate choice of these models may reduce the overall performance of the algorithm. In this paper, we propose a novel SR algorithm which utilizes a complex wavelet-based de-noising approach. The proposed algorithm uses a bivariate Laplacian mixture model in a sparseness constraint to regularize the SR inverse problem. Our results show that the 3D MRI volumes reconstructed using this approach have quality superior to volumes produced by the best previously proposed approaches.

Index Terms— Magnetic Resonance Imaging, Super Resolution, Bivariate Laplacian Mixture Model, Wavelet Regularization.

1. INTRODUCTION

MRI is an imaging technique used in medical settings for producing images of parts of the human body for clinical purposes. It is a non-invasive method and is harmless to the patient because it uses strong magnetic fields and non-ionizing radiation in the radio frequency range. In multi-slice MRI, multiple 2D slices are captured to form a 3D volume of the desired body part. However, the resolution in the slice direction is usually reduced to allow faster acquisition times and to reduce the amount of noise in each 2-D slice. As a result of this resolution reduction some valuable information may be lost [1]. To deal with this problem, a number of image SR techniques have been used as a post-processing method to improve the quality of the low resolution images.

The basic idea underlying all SR algorithms is the combination of a number of low resolution (LR) blurred noisy images to produce an SR image. In recent years, many SR algorithms have been proposed [2]. Within this wide area of study, medical image super resolution has emerged as a particularly active field and the use of regularization has been shown to improve the performance of SR algorithms. Recently a spatial-domain regularization for SR was proposed in [3]. This regularization method, known as directional bilateral total variation (DBTV), applied a directionally adaptive weighting on the tradition total variation regularization approach. Schilling et. al. introduced a multi-stack approach in [4] where a number of LR MRI volumes were captured at different orientations and combined to produce an SR volume using the simultaneous additive reconstruction technique (SART). They showed that capturing the LR images at different orientations produced superior results than a parallel stack approach where the LR images were captured at different positions along the slice-select direction only.

In this paper, we present a new SR algorithm for MRI volumes. In our proposed approach we adopt wavelet-domain regularization and use the bi-variate Laplacian mixture model (BLMM) de-noising approach proposed in [5]. Our results show that this algorithm produces SR volumes with quality superior to volumes produced by the best previously proposed approaches.

The remainder of the paper is organized as follows: In Section 2, we explain the general approach used in our SR approach. Our proposed extensions to this approach for generating a super-resolution volume are presented in Section 3. Section 4 is devoted to the experimental evaluation of the algorithms tested and finally our conclusions are presented in section 5.

2. IMAGE DE-CONVOLUTION USING WAVELET-BASED REGULARIZATION

The main objective of image de-convolution algorithms is to find the solution of an ill-conditioned equation of the form [6]:

$$\mathbf{y} = \mathbf{H}\mathbf{x}_o + \mathbf{b} \tag{1}$$

where $\mathbf{x}_o$ is a vector of pixel values from the original image, $\mathbf{y}$ is a vector of pixel values from the observed blurry image, $\mathbf{H}$ represents a convolution (i.e. block-circulant) matrix that

approximates the blurring function and **b** is a vector representing white noise with zero mean and variance σ^2.

A solution to this minimization problem can be explained by first considering the case when the noise term **b** is set to zero. The cost function for this case is given by:

$$J(\mathbf{x}) = \|\mathbf{y} - \mathbf{H}\mathbf{x}\|_2^2 \tag{2}$$

The solution that minimizes this cost function is given by the classic Landweber iteration as follows:

$$\mathbf{x}^{n+1} = \mathbf{x}^n + \tau\, \mathbf{H}'(\mathbf{y} - \mathbf{H}\mathbf{x}^n) \tag{3}$$

where τ is a fixed step size. Now consider the case when no blurring occurs but noise is present in the system. A cost function that aims to minimize the ℓ_1 norm of the wavelet coefficients of **x** can be used to solve this problem as follows:

$$J(\mathbf{x}) = \|\mathbf{y} - \mathbf{x}\|_2^2 + \lambda\|\mathbf{W}\mathbf{x}\|_1 \tag{4}$$

where **W** represents the wavelet transform operation and the regularization parameter λ balances the contribution of both terms. The optimal solution for this function is provided by soft-thresholding of the wavelet coefficients of **x** as follows:

$$\mathbf{x} = \mathbf{M}\Phi(\mathbf{W}\mathbf{x}, \lambda/2) \tag{5}$$

where **M** denotes the inverse wavelet transform operation and the function $\Phi(w,t)$ is the soft-thresholding function as defined in [7].

Daubechies et al. proposed a solution to the problem of estimating the value of $\mathbf{x}_o$ when both blurring and noise are present by using the thresholded-Landweber algorithm [8]. This approach consists of first performing the Landweber iteration with step-size τ:

$$\mathbf{z}^n = \mathbf{x}^n + \tau\, \mathbf{H}'(\mathbf{y} - \mathbf{H}\mathbf{x}^n) \tag{6}$$

and then performing the wavelet domain de-noising operation:

$$\mathbf{x}^{n+1} = \mathbf{M}\Phi(\mathbf{W}\mathbf{z}^n, \tau\lambda/2) \tag{7}$$

Vonesch and Unser [7] improved this approach by developing the fast thresholded-Landweber (FTL) de-blurring algorithm. In their approach, the original thresholded-Landweber operation is performed on a wavelet sub-band basis using different step-sizes for each sub-band as follows:

For each sub-band *j*, perform the Landweber iteration:

$$\mathbf{z}_j^n = \mathbf{W}_j\left(\alpha_j \mathbf{x}^n + \mathbf{H}'(\mathbf{y} - \mathbf{H}\mathbf{x}^n)\right) \tag{8}$$

where $\mathbf{W}_j$ represents the wavelet transform operation for every sub-band *j* and α_j is the sub-band emphasis factor defined by Vonesch and Unser in [7]. Then perform the wavelet domain soft-thresholding de-noising operation on each sub-band and apply the inverse wavelet transform to find the updated version of **x**:

$$\mathbf{x}^{n+1} = \sum_{j \in S} \mathbf{M}_j \Phi(\mathbf{z}_j^n, \tau_j \lambda/2) \tag{9}$$

3. SUPER RESOLUTION USING A BLMM CONSTRAINT

The MRI image capturing process can be represented by a linear filter followed by down-sampling and the addition of noise of zero mean and variance σ^2. Hence, the objective of our super resolution algorithm is to find the solution of an ill-conditioned equation of the form:

$$\mathbf{y}_i = \mathbf{D}\mathbf{H}\mathbf{F}_i\mathbf{x}_\mathbf{o} + \mathbf{b}_i \tag{10}$$

where $\mathbf{y}_i$ is a vector of pixel values from the *i*th observed image, $\mathbf{F}_i$ is a warping matrix used to register $\mathbf{x}_\mathbf{o}$ with $\mathbf{y}_i$ and **D** is a sub-sampling matrix.

In our proposed approach we extended the FTL algorithm to include multiple observed images and sub-sampling in the image acquisition model. To accommodate this extension we modify the Landweber iteration in (8) as follows:

$$\mathbf{z}_j^n = \mathbf{W}_j\left(\alpha_j \mathbf{x}^n + \sum_{i=1\ldots k} \mathbf{F}_i^T \mathbf{H}^T \mathbf{D}^T (\mathbf{y}_i - \mathbf{D}\mathbf{H}\mathbf{F}_i\mathbf{x}^n)\right) \tag{11}$$

A component wise wavelet based de-noising operation is then performed on every sub-band and the inverse wavelet transform is applied to find the updated version of **x** using the following equation:

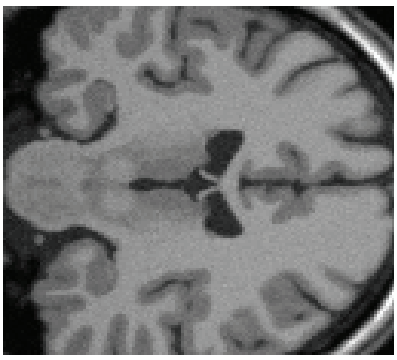
$$\mathbf{x}^{n+1} = \sum_{j \in S} \mathbf{M}_j \left(\frac{(\hat{\mathbf{w}}_j^2 + \varepsilon^2)\mathbf{z}_j^n}{\alpha_j(\hat{\mathbf{w}}_j^2 + \varepsilon^2) + \sigma^2} \right) \tag{12}$$

This de-noising approach was shown by Zhang and Kingsbury in [9] to be equivalent to replacing the ℓ_1 norm sparseness constraint in equation (4) with a sparseness constraint that is closer to the ℓ_0 norm. In equation (12) $\hat{\mathbf{w}}_j$ is calculated using the Laplacian mixture model de-noising approach defined in [5] and ε is a regularization parameter. We use this Laplacian mixture model approach as it can accurately characterize both the heavy tail distribution and the inter-scale properties of wavelet coefficients.

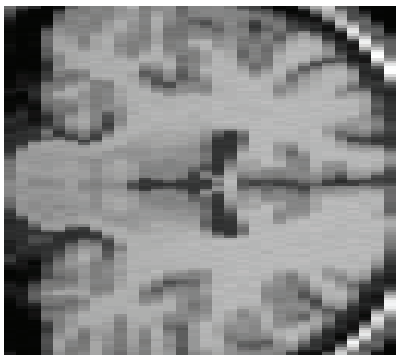
4. PERFORMANCE EVALUATION

4.1 Experimental Procedure

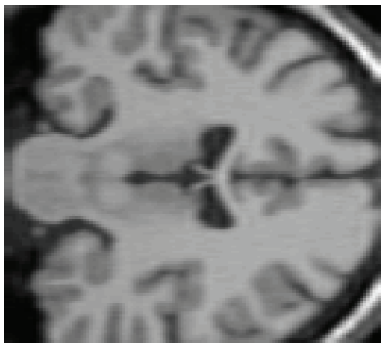
We conducted experiments to evaluate the performance of the proposed SR algorithm for the purposes of increasing the resolution of 3D MRI volumes. The simulated input data used was a 3D MRI volume from the Brain Web Simulated Database [10]. Figure 1(a) shows a slice from the original volume. We compared the performance of our proposed algorithm with a SR algorithm based on DBTV [3] as well as the SART [4] approach as these were found to be the best performing of the recently proposed SR approaches. The wavelet transform used in our algorithm was a 3D dual tree



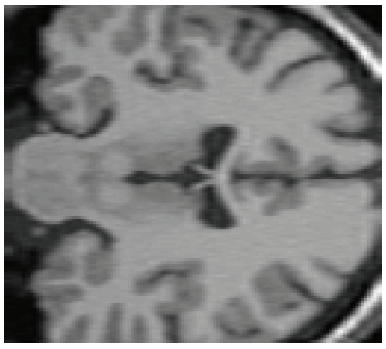
(a) Original slice



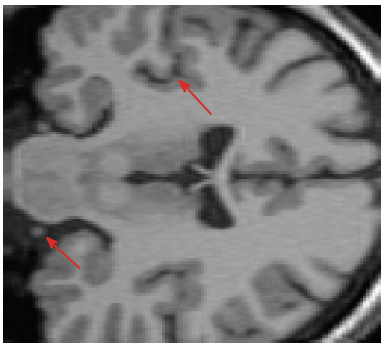
(b) LR slice



(c) SR with DBTV



(d) SR with SART



(e) Proposed algorithm

Figure 1. One slice from (a) The original volume, (b) One of the LR input volumes, (c) The output volume for the SR with DBTV algorithm, (d) The output volume from the SR with SART algorithm and (e) The output volume for the proposed SR with BLMM constraint algorithm.

complex wavelet transform (DTCWT) as this transform was found in [9] to be superior to the Shannon wavelet.

In our experiment, we took a 3D volume with 1mm resolution in the slice-select direction and rotated it about all three axes simultaneously by 1, 2, 3 and 4 degrees. Each of these rotated volumes was then sub-sampled by a factor of 4 in the slice select direction. Blur and noise were also added to simulate the effect of the image acquisition process. We used the multi-stack approach of capturing LR volumes at different orientations as this technique as found give better performance than the parallel stack approach in [4]. In the MRI acquisition process the point spread function (PSF) is defined by the slice excitation profile function. In our experiment, the slice profile functions are approximated by Gaussian functions with the full-width half maximum (FWHM) distance set to the selected slice thickness.

4.2 Experimental Results

Figure 1(b) shows a slice from one of the sub-sampled, blurred and noisy input volumes, Figure 1(c) shows one slice from the output of the SR with DBTV algorithm, Figure 1(d) shows one slice from the output of the SR algorithm using SART and Figure 1(e) shows one slice from the output of our proposed SR algorithm using a BLMM constraint. These images clearly show the improved subjective quality of the output of our proposed algorithm. The arrows in Figure 1(e) indicate areas of improved resolution for the proposed algorithm.

For a practical measure of the quality of the estimated super resolution volume, we use a measure of the accuracy of a 3D segmented region produced using the estimated volumes. We call this measure the segmentation difference (SD) and it is defined as the percentage of voxels which are segmented differently for the estimated volume compared to the segmentation produced using the original volume. For a more standard evaluation we also compared the PSNR between the output volume and the original volume as a function of the iteration number.

The results of these comparisons are shown in Figure 2 and Table I. Figure 2(a) shows a plot of the segmentation difference at each iteration when the white matter from the estimated SR volumes was segmented and compared to the segmented white matter from the original volume. Figure 2(b) shows the curves for PSNR of the estimated volume compared to the original volume for each SR technique. These results clearly show that the performance of our proposed SR algorithm using a BLMM constraint is superior

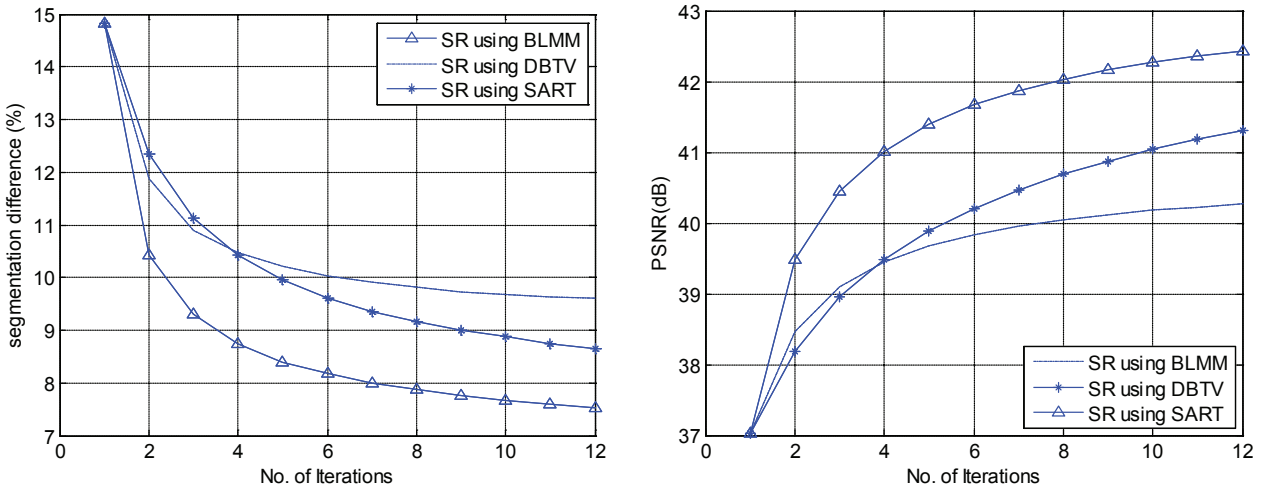


Figure 2. (a) The segmentation difference and (b) The PSNR for the SR algorithms using DBTV, SART and the proposed BLMM model constraint.

to the alternative SR with DBTV and SR with SART algorithms. The PSNR values when compared with the original volume after the final iterations of each algorithm are shown in Table I.

TABLE I. FINAL PSNR VALUES FOR EACH ALGORITHM

Algorithms	SR with DBTV	SR using SART	SR using BLMM constraint
PSNR (dB)	40.26	41.30	**42.43**

5. CONCLUSIONS

In this paper, we presented a SR algorithm using a BLMM to approximate the wavelet sparseness constraint. The algorithm takes several multi-slice volumes of the same anatomical region captured at different orientations and combines these low-resolution volumes together to form a single 3D volume with much higher resolution in the slice direction. Our results show that using this constraint provides superior performance to the alternative SR with DBTV and SR using SART approaches both visually and quantitatively.

6. REFERENCES

[1] H. Greenspan, "Super-Resolution in Medical Imaging," *The Computer Journal,* vol. 52, pp. 43-63, January 1, 2009 2009.

[2] P. Sung Cheol, P. Min Kyu, and K. Moon Gi, "Super-resolution image reconstruction: a technical overview," *Signal Processing Magazine, IEEE,* vol. 20, pp. 21-36, 2003.

[3] A. Ben-Ezra, H. Greenspan, and Y. Rubner, "Regularized super-resolution of brain MRI," in *Biomedical Imaging: From Nano to Macro, 2009. ISBI '09. IEEE International Symposium on*, 2009, pp. 254-257.

[4] R. Z. Shilling, et al., "A Super-Resolution Framework for 3-D High-Resolution and High-Contrast Imaging Using 2-D Multislice MRI," *Medical Imaging, IEEE Transactions on,* vol. 28, pp. 633-644, 2009.

[5] H. Rabbani, R. Nezafat, and S. Gazor, "Wavelet-Domain Medical Image Denoising Using Bivariate Laplacian Mixture Model," *Biomedical Engineering, IEEE Transactions on,* vol. 56, pp. 2826-2837, 2009.

[6] M. Bertero and P. Boccacci, "Introduction to Inverse Problems in Imaging," Taylor & Francis, 1998.

[7] C. Vonesch and M. Unser, "A Fast Thresholded Landweber Algorithm for Wavelet-Regularized Multidimensional Deconvolution," *Image Processing, IEEE Transactions on,* vol. 17, pp. 539-549, 2008.

[8] I. Daubechies, M. Defrise, and C. De Mol, "An iterative thresholding algorithm for linear inverse problems with a sparsity constraint," *Communications on Pure and Applied Mathematics,* vol. 57, pp. 1413-1457, 2004.

[9] Y. Zhang and N. Kingsbury, "Image deconvolution using a Gaussian Scale Mixtures model to approximate the wavelet sparseness constraint," in *Proceedings of the 2009 IEEE International Conference on Acoustics, Speech and Signal Processing*: IEEE Computer Society, 2009, pp. 681-684.

[10] D. L. Collins, et al., "Design and construction of a realistic digital brain phantom," *Medical Imaging, IEEE Transactions on,* vol. 17, pp. 463-468, 1998.