A LOGNORMAL MULTIPLICATIVE CASCADE MODEL FOR MULTIFRACTAL CHARACTERIZATION OF SKIN ULTRASOUND IMAGES

M. Djeddi, Z. Ouksili, H. Batatia

University of Toulouse, IRIT - INPT, 2 rue Charles Camichel, F31071 Toulouse

ABSTRACT

Biological tissue characterization requires adequate models of the data. Research in ultrasound image modeling has mainly focused on statistical methods. Some authors have addressed the fractal properties of such images using fractional Brownian motion model. However, recent studies have shown that skin ultrasound signals have multifractal properties. This paper proposes a lognormal multiplicative cascade model to explain this multi-fractality. Experimental results on real images show the validity of the model supported by excellent KS tests. The parameters of the model are proposed as indicators to characterize underlying tissues. Successful application to distinguish melanoma from healthy dermis is presented.

Index Terms— Multiplicative cascade, lognormal distribution, ultrasound images, multifractal analysis, skin characterization.

1. INTRODUCTION

Diagnosing skin lesions is mainly based on visual surface examination. Gold standard is provided by anatomopathological analysis. Current practices lead to large numbers (80%) of unnecessary surgery. Analyzing in-vivo ultrasound images allows reducing this percentage. This requires robust automatic segmentation and characterization tools. Ultrasound signals backscattered from biological tissues are usually described by the point scattering model. This model states that tissues are homogeneous media with randomly distributed micro-structures, named scatterers, that cause backscattering. The scatterers cannot be resolved individually in a single resolution cell leading to interference in backscattered echoes that result in the phenomenon called speckle. Based on this model, various research works proposed statistical distributions to describe radiofrequency (RF) signals and their envelop [1, 2, 3]. Tissue characterization is achieved by linking statistical parameters with scatterers properties, such as density and size. Recently in [4], authors proposed a generalized Rayleigh model for skin ultrasound images and showed that scatterers have power law statistics. Fractal analysis has become a classic technique to model scale invariance of signals. Fractal parameters are often used to classify textures [5]. Other authors proposed stochastic models to represent images. Chainais [6] presented a multifractal model for ultraviolet images of the sun and used it to simulate images of arbitrary resolution. Guyot et al. studied the relationship between fractional Brownian motion (fBm) and optical speckle fields [8] and proposed fractal characterization of skin psoriasis. In ultrasound images, fractal modeling is usually based on fBm [7]. The Holder exponent is used to evaluate the statistical complexity of structures. Chen et al [9] used fractal dimension estimated from an fBm to study breast lesions in ultrasound images. They proposed an unsupervised method to classify malignant and benign tumors. Other works assessed the potential of fractal dimension for tissue characterization [10, 11]. More recent work has shown that skin ultrasound images have multifractal properties that better capture the complexity of their structures [12]. This paper presents a lognormal multiplicative cascade model for skin ultrasound images that explains the empirically established multifractality. Parameters of this model are shown to discriminate melanoma and healthy dermis.

This work has been conducted under the CAMM4D program funded by the French FUI and the Midi-Pyrenees Regional Council.

2. PROBLEM STATEMENT

Let $x(t)$ be a radio-frequency ultrasound signal backscattered from the skin. Let $y(t)$ be its Hilbert transform. The B-mode echographic image $u(t)$ is the envelop of the analytic signal:

$$u(t) = |x(t) + jy(t)| \tag{1}$$

It is widely accepted that $x(t)$ derives from the point scattering model. Assuming that the diffusion function $p(t)$ is identical in all points and that there is no multiple scattering, $x(t)$ can be considered as the sum of random echoes backscattered by N scattering points, proportionally to their size:

$$x(t) = \sum a_i p(t - \tau_i) \tag{2}$$

where a_i is the cross-section of the ith scatterer and τ_i its position. Assuming that N is large and a_i are i.i.d random variables, by applying the central limit theorem, $x(t)$ can be written as the product of the reflectivity and a multiplicative noise:

$$x(t) = n(t)\sqrt{v(t)} \tag{3}$$

$n(t)$ is called fully developed speckle. If scatterers are i.i.d and uniformly distributed, $x(t)$ is Gaussian, $x(t) \sim N(0, \sigma^2)$,

and the reflectivity is constant, $v(t) = \sigma^2$. In this case, the envelop $u(t)$ follows a Rayleigh distribution:

$$u(t) \sim P(u) = \frac{u}{2\sigma^2} exp(-\frac{u}{2\sigma^2}) \quad (4)$$

This flexible framework has often been used to analyse ultrasound images given the linearity of the Gaussian model and its invariance by convolution. In this context, Guyot et al. [8] proposed to model the speckle of skin optical images using a fractional Brownian motion. The authors used the Holder exponent and the saturation of variance to characterize tissues. However, similarly to ultrasound images, speckle of optical images have a Rayleigh distribution. In addition, several works have shown that RF ultrasound signals are non Gaussian and presents heavy tailed statistics [4]. The fBm model is therefore inappropriate to describe the fractality of these images. Simultaneously, in [12], the authors showed empirically that skin ultrasound B-mode images have multifractal properties. In this paper, we propose a stochastic model to represent the speckle of the skin. This model provides multifractal indicators to characterize skin tissues, such as melanoma and dermis.

3. PROPOSED MODEL

3.1. Lognormal Cascade

Multiplicative cascades of Mandelbrot [13] are a family of multifractal processes proposed to explicate scale invariance. Their iterative construction is based on a dyadic grid $[x_{j,k} = ((k + 1/2)2^{-j}, 2^{-j})]$, $j \in N$, $k \in N$ and random multipliers $W_{i,j} > 0$ associated to the grid nodes $(x_{j,k}, r_{j,k})$ located at the resolution $r_{j,k}$. For scales in the range $]0, 1]$, the result $Q_r(x)$ of the cascade at the resolution $r > 0$ can be written [5]:

$$Q_r(x) = \prod_{j=1...J,k:t\in[2^j k, 2^j(k+1)]} W_{j,k} \quad (5)$$

where $r = 2^{-J}$ after J iterations. In this work, we propose to model speckle in B-mode ultrasound images of skin as a multiplicative cascade with multipliers W_{jk} distributed as lognormal:

$$f_{W_{jk}}(x; \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} exp((-\frac{log2(x) - \mu}{2\sigma})^2) \quad (6)$$

where μ et σ are the mean and standard deviation of the log of multipliers. This choice is justified, on one hand, by the fact that the scaling law exponents of the lognormal cascade can be expressed as the sum of two terms:

$$\zeta(q) = qH + \tau(q) \quad (7)$$

where q is the order of the statistical moments and $\tau(q)$ is a concave non-linear function defined by:

$$\tau(q) = \frac{\sigma^2}{2} q^2 - \mu q; \tau(0) = \tau(1) = 0 \quad (8)$$

The parameter H represents the linear trend of $\zeta(q)$. The multifractal nature of the image will be governed by $\tau(q)$, and precisely by the parameters μ and σ. The second argument in choosing lognormal multipliers is the existence of an analytical expression of the multifractal spectrum:

$$D(h) = -\frac{h + \frac{\mu}{log(2)}}{\frac{2\sigma^2}{log(2)}} \quad (9)$$

where h is the Holder exponent. Another advantage of this model is the direct interpretation of the parameters μ et σ in relation to the spectrum. Precisely, μ is its scale parameter and σ a shape parameter. These two parameters are sufficient in describing the mode of the spectrum, which is the most interesting aspect of the multifractal analysis. Finally, we note that the lognormal law is more adequate to the heavy tailed nature of the statistics of ultrasound than Gaussian or exponential laws.

3.2. Parameter estimation

In order to estimate the parameters of the model, we perform a multifractal analysis following the wavelet transform maxima method [12, 5]. Let $T_u(a, t)$ be the maximum wavelet coefficients describing the image $u(t)$ at a location t and an analysis scale a. We define the structure functions for an order q, $S_q(j)$:

$$S_q(j) = \frac{1}{N_j} \sum_{i=1}^{N_j} |T_u(a, t)|^q \quad (10)$$

Here, N_j denotes the number of $T_u(a, t)$ available at scale 2^j and $-2 \leq q \leq 2$. The scaling law exponents $\zeta(q)$ are estimated, from the data, so that $S_q(j)$ have power law behavior:

$$S_q(j) \propto 2^{j\zeta(q)} \quad (11)$$

The estimators of these exponents are given by:

$$\widehat{\zeta}(q) = \liminf_{j \longrightarrow 0} ln \frac{S_q(j)}{j} \quad (12)$$

The singularity spectrum can subsequently be obtained by Legendre transformation of the estimated exponents:

$$\widehat{D}(h) = \min_{q\neq 0}(1 + qh - \widehat{\zeta}(q)) \quad (13)$$

The Holder exponents h are estimated as follows:

$$\widehat{h}(q) = \frac{d\zeta(q)}{dq} \quad (14)$$

Finally, the parameters μ et σ are estimated from the spectrum as:

$$\mu = h(q = 0) * log(2) \quad (15)$$

$$\sigma^2 = (h(q_{max} - h(q_{min}) * log(2) \quad (16)$$

4. EXPERIMENTATION

Several experiments have been conducted to evaluate the performance of the lognormal cascade to represent skin ultrasound images. The corpus of images was acquired with an ATYS Dermcup 3D system having a single-element focalized 25 MHz probe sampled at 100 MHz (Fig. 1). Images have been annotated by clinicians to roughly indicate the melanoma and the borders of skin layers.

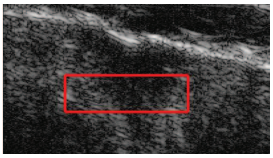


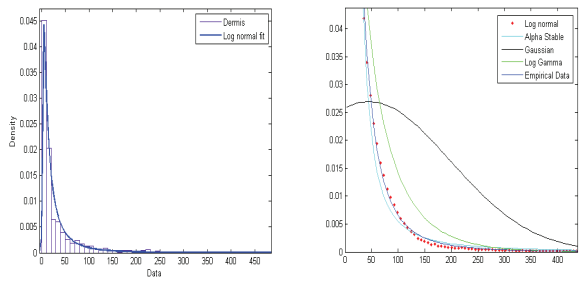
Fig. 1. Ultrasound image from the corpus with a region of interest.

4.1. Lognormal goodness of fit

The first experiment concerned the goodness of fit of the lognormal distribution to the data histogram (Fig. 2). In order to emphasize the fit of the tail, figure 2.b shows a zoom on the tail of the density function. To assess the relevance, Gaussian, log Gamma and alpha-stable laws have been added. We notice that the histogram has a heavy tail which is correctly fitted by the lognormal law; although alpha-stable gives a better fit.

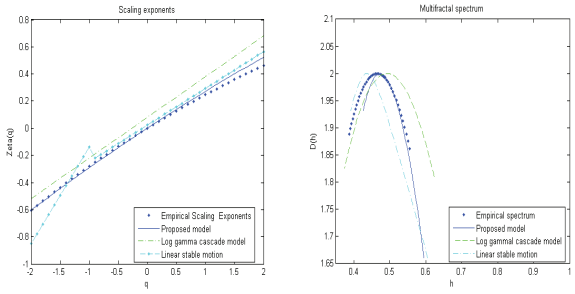
4.2. Performance of the cascade model

Another series of experiments have been conducted to evaluate the performance of the cascade model to fit multifractal properties of the data. Figure 3.a shows the scaling laws function $\zeta(q)$ for lognormal, log Gamma, linear stable motion, and the exponents obtained from the multifractal analysis. Figure 3.b shows the multifractal spectrum $D(h)$ estimated for the three models and the empirical spectrum. We notice in both figures that the lognormal cascade fits better the multifractal properties of the data, particularly at the mode. We evaluated quantitatively the goodness of fit using Kolmogorov-Smirnov (KS) test. Figure 4 presents the results. These confirm the better fit of the lognormal cascade.



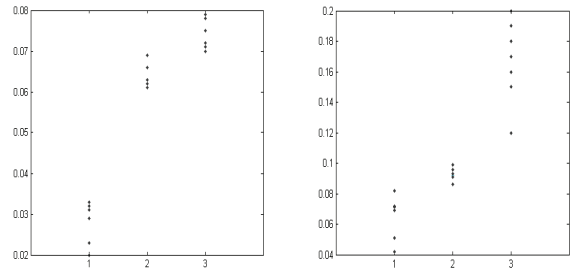
(a) (b)

Fig. 2. Fit of distributions to data: (a) log nomal .(b)Zoom on the tail.



(a) (b)

Fig. 3. (a)Scaling law exponents. (b)spectrum for different cascades.

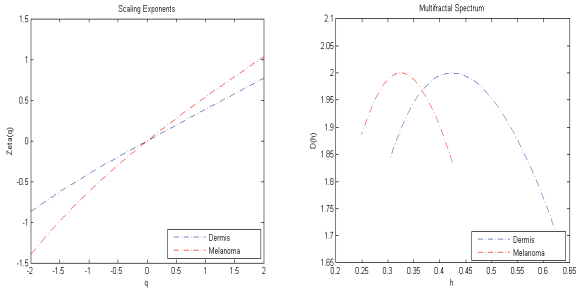


(a) (b)

Fig. 4. KS tests (1.Log normal,2.Linear stable motion,3.Log gamma): a)scaling law exponents, b)spectrum.

4.3. Multifractal characterization of melanoma

The proposed model allows describing tissues using a parameter vector robust to speckle and easy to interpret. In order to establish the ability of the parameters to discriminate tissues, we compared their values for healthy dermis and melanoma. Figure 5.a shows the scaling law exponents $\zeta(q)$ for the dermis and the melanoma. The multifractal spectrum $D(h)$ are presented in figure 5.b. We notice that both exponents and spectrum are clearly distinct for the two tissues. Table 1 illustrates the estimated parameters (μ, σ^2) for the lognormal model for several images. It appears that the melanoma, which has a high contrast, gives a small μ and σ. Inversely, μ is higher for the dermis. Although the esti-



(a) (b)

Fig. 5. (a)Scaling law exponents $\zeta(q)$. (b)spectrum $D(h)$ for dermis and melanoma

mation of the parameters is not invariant by rotation, due to the discrete wavelet transformation, in our case these parameters are robust to rotation. This is due to the fact that ultrasound images are vertical slices of the tissue. The layered structure of the skin gives similarly structured images whatever their orientation is, as long as the probe is normal to the surface. We have also evaluated the precision and the power

	Dermis		Melanoma	
$Images$	μ	σ^2	μ	σ^2
1	0.4359	0.2439	0.2486	0.1542
2	0.3787	0.2965	0.2685	0.1982
3	0.4043	0.2159	0.1603	0.1739
4	0.3825	0.2647	0.2687	0.1343
5	0.3950	0.2099	0.2387	0.1697
6	0.3843	0.2353	0.2070	0.1821
7	0.3871	0.2289	0.2761	0.1958
8	0.3908	0.2238	0.2377	0.1549

Table 1. Estimated parameters μ and σ .

[14] of the estimators for different values for the risk threshold $\alpha \in \{0.05, 0.1, 0.025\}$. Precision has been evaluated by the degree of significance α_s and power by the β test. Table 2 shows the results. Notice that low α_s and high β indicate respectively better precision and power.

5. CONCLUSION

In this paper we proposed a multiplicative lognormal cascade to represent ultrasound images of human skin. This model generalizes the classic fractional Brownian motion model usually used for these images. The proposed model is based on lognormal multipliers that bring different advantages, namely the existence of analytic expression of the multifractal spectrum, the quadratic form of the function of scaling laws, and the existence of two simple parameters that has direct interpretation. We presented estimators for the parameters using multifractal analysis using wavelet leaders. Several experiments, on real in-vivo images, have been presented to validate and evaluate the model. The lognormal law has been fitted to the data. The function of scaling laws and the spectrum of the model have been fitted to those of the empirical data and compared with other multiplicative cascades. The KS test gave an excellent goodness of fit. We proposed to characterize skin tissues using the pair of parameters of the model. This parameter vector has proven efficient in discriminating melanoma from healthy dermis. Future work aims at developing a segmentation method based on this model.

	$Percentil$			$Normal$		
α	0.05	0.15	0.25	0.05	0.15	0.25
Dermis	0.05	0.11	0.20	0.03	0.12	0.21
(α, β)	0.76	0.89	0.84	0.89	0.81	0.88
Melanoma	0.04	0.10	0.18	0.04	0.11	0.23
(α, β)	0.62	0.56	0.77	0.85	0.89	0.76

Table 2. Precision and power of the estimators.

6. REFERENCES

[1] R. F. Wagner, M. F. Insana, and D. G. Brown (1987). Statistical properties of radio-frequency and envelope-detected signals with applications to medical ultrasound. *J. Opt. Soc. Am. A*, vol. 4, no. 5, pp. 910922.

[2] P. Shankar (2000). A general statistical model for ultrasonic backscattering from tissues. *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control*, vol. 47, no. 3, pp. 727736

[3] B. I. Raju (2002). *High frequency ultrasonic characterization of human skin in vivo.* Ph.D. dissertation, Massachusetts Institute of Technology, Cambridge, MA.

[4] M. Pereyra, H. Batatia (2010). An alpha-stable model for ultrasound speckle statistics in skin. *IEEE engineering in Medecine and Biology Conference*, pp. 4715-4718.

[5] H. Wendt, P. Abry, S. Jaffard, H. Ji and Z.Shen (2009). Wavelet leader multifractal analysis for texture classification. *IEEE International Conference on Image Processing (ICIP)*, pp. 3829-3832.

[6] P. Chainais, V. Delouille and J. F. Hochedez (2007). Vers un modle sous-pixel des images de Soleil calme dans l'extreme ultraviolet. *in Proc. GRETSI'2007* , France, pp. 705-708.

[7] R. Jennane, R. Harba and G. Jacquet (2001). Analysis methods for fractional brownian motion: theory and comparative results. *Traitement du Signal*, vol. 18, num. 5-6, pp. 419-436.

[8] S. Guyot, M. C. Peron and E. Delchelle (2004). Spatial Speckle characterization by Brownian analysis. *Physical Review*, vol. 70 , Issue 4, pp. 046618.1-046618.8.

[9] D. R. Chen, R. F. Chang, C. J. Chen, M. F. Ho, S. J. Kuo, S. T. Chen, S. J. Hung and W. K. Moon (2005). Classification of breast ultrasound images using fractal Feature. *Journal of Clinical Imaging*, vol. 29, pp. 235-245.

[10] W. L. Lee, Y. C. Chen and K. S. Hsieh (2003). Ultrasonic liver tissues classification by fractal feature vector based on M-band wavelet transform. *IEEE Transaction on Medical Imaging*, Vol. 22, Issue 3, pp. 382-392.

[11] W. L. Lee, Y. Chang Chen, Y. Cheng Chen and K. S. Hsieh (2005). Unsupervised segmentation of ultrasonic liver images by multiresolution fractal feature vector. *Information Sciences*, vol. 175, Issue 3, pp. 177-199.

[12] M. Djeddi, A. Ouahabi, H. Batatia, A. Basarab and D. Kouam (2010). Discrete wavelet for multifractal texture classification: application to medical ultrasound. *Ieee International Conference on Image Processing (ICIP)*, pp. 637-640.

[13] J. P. Kahane, J. Peyrire (1976). Sur certaines martingales de Benoit Mandelbrot. *Adv. Math*, vol. 22, pp. 131145.

[14] H. Wendt, P.Abry. Multifractality Tests Using Bootstrapped Wavelet Leaders . *Signal Processing, IEEE Transactions on* , vol. 55, pp. 4811 - 4820