LOCAL PHASE APPROACHES TO EXTRACT BIOMEDICAL NETWORKS

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ABSTRACT

Many biomedical applications require detection of curvilinear networks in images, and would benefit from automatic or semiautomatic segmentation to allow high-throughput measurements. Here we discuss a contrast independent approach to identify curvilinear structures based on oriented phase congruency, the Phase Congruency Tensor. We show that the proposed approach is largely insensitive to intensity variations along the curve, and provides successful detection within noisy regions. Moreover, we demonstrate that the proposed approach may be used in a wide range of curvilinear and non-curvilinear feature enhancement and detection methods, particularly where tensor representation of the image is explored. The performance of the Phase Congruency Tensor-based methods is evaluated by comparing it with state-of-the-art intensity-based methods on both synthetic and real images of biomedical networks.

Index Terms— Bioimage informatics, Phase Congruency Tensor, curvilinear structure, branching structure, live-wire tracing, feature detection, vector field, directional statistics, blob detector, anisotropic diffusion filtering, coherence enhancing, biomedical networks.

1. INTRODUCTION

Networks in the natural world occur wherever there is a need for a distributary system. The form of these natural networks is a physical manifestation of their responses to changes in their internal and external contexts. By studying the form, development and dynamics of a natural network in relation to these contexts, we can interpret its dynamic portrait, to find a story of how the network became and is becoming the system that it is now [1].

Robust and efficient enhancement, segmentation, analysis and modeling of curve-like networks is a common problem in biomedical image informatics. In particular, the detection of curvilinear features is commonly affected by variations of intensity contrast within the image, due e.g. to variations in illumination, bias field effects in MRI or varying uptake of contrast agents. This is especially important in detection and analysis of biomedical networks where the network connectivity is critical. New revolutions in the field can be made possible by developing the brightness- and contrast-invariant bioimage informatics technologies to extract and exploit information from imaging data so as to achieve new fundamental biological insights and understanding, as well as developing possible therapeutic strategies in medical applications.

In this paper, we discuss a recently introduced and developed concept for curvilinear feature detection, called the Phase Congruency Tensor (PCT) [2]. This approach expands the idea of phase congruency to incorporate the local structure of the image at different orientations, thus keeping the contrast-invariant property of phase-based methods while allowing the detection of structures with specific shapes. Through the use of its eigenvalues and eigenvectors the PCT can be used to reduce contrast dependency in many methods for detection and analysis of curvilinear structures observed in biomedical images. Its use in exchange for state-of-the-art intensity-based methods such as: vesselness, neuriteness, live-wire tracing and anisotropic diffusion filtering is illustrated in the following Sections. Obtained results show that the PCT approach is largely insensitive to intensity variations along the curve, and provides successful enhancement and detection within noisy regions. The performance of the PCT-based methods was evaluated by comparing them with equivalent intensity-based methods on both synthetic and real biological images.

2. PHASE CONGRUENCY TENSOR

In general, local tensor-based representations can be produced by combining the outputs from polar separable quadrature filters, applied on several orientations.While tensor representations can be built on purely intensity-based filters, these have the downside of being sensitive to changes in image contrast. Methods based on local phase have been proposed as a contrast-independent alternative for feature detection. In particular, phase congruency [3] is based on the concept that salient features have similar values of local phase when observed at different scales. Here we exploit the idea that phase congruency values are high in the direction perpendicular to the structure, while they remain close to zero in the direction parallel to the structure. More importantly, the values of

This work was supported by grants from the BBSRC, EPSRC and NERC, UK.

phase congruency are minimally affected by contrast changes. Thus, for a given image $I(\mathbf{p})$, a set of scales $\{s\}$, a set of orientations $\{o\}$ and a given set of phase congruency measures $PC_o(\mathbf{p})$ (for each orientation o) [3], the Phase Congruency Tensor takes the following form [2]:

$$T_{PC} = \sum_o PC_o(\mathbf{p})(\mathbf{n}_o\mathbf{n}_o^T - \alpha\mathbb{I}) \quad (1)$$

where $\mathbf{p} = [x, y]^T$ represents pixel location, $\mathbf{n}_o$ is the normalized orientation vector in the direction o, $\alpha = \frac{1}{m-1}$, with m being the dimensionality of the image I and $\mathbb{I}$ is the identity tensor.

The eigen-decomposition of the tensor T_{PC} results in $\lambda_{1,2}$, $\lambda_1 \geq \lambda_2$ and $t_{1,2}$ for the eigenvalues and eigenvectors respectively.

3. FEATURE ENHANCEMENT

3.1. PCT Vesselness and Neuriteness

Piecewise curvilinear segments can be detected by analyzing the relations between eigenvalues and eigenvectors of the locally-calculated Hessian. Two of the ways in which this relations have been tackled in the literature, are measures of coherence of curvilinear structure named vesselness [4] and neuriteness [5].

For a given scale σ, the Hessian matrix $H_\sigma(\mathbf{p})$ of the image $I(\mathbf{p})$ is computed, and the vesselness

$$V_\sigma = e^{-\frac{\lambda_{\sigma,1}^2}{2\beta^2\lambda_{\sigma,2}^2}}\left(1 - e^{-\frac{\lambda_{\sigma,1}^2+\lambda_{\sigma,2}^2}{2c^2}}\right) \quad (2)$$

and the neuriteness

$$N_\sigma = \begin{cases} \frac{\lambda_\sigma}{\lambda_{\sigma,min}}, & \text{if } \lambda_\sigma < 0 \\ 0, & \text{if } \lambda_\sigma \geq 0 \end{cases} \quad (3)$$

$$\lambda_{\sigma,1}{}' = \lambda_{\sigma,1} + \alpha\lambda_{\sigma,2} \quad (4)$$
$$\lambda_{\sigma,2}{}' = \lambda_{\sigma,2} + \alpha\lambda_{\sigma,1} \quad (5)$$
$$\lambda_\sigma = \max\left(|\lambda_{\sigma,1}{}'|, |\lambda_{\sigma,2}{}'|\right) \quad (6)$$
$$\lambda_{\sigma,min} = \min_{\mathbf{p}\in I}(\lambda_\sigma) \quad (7)$$

measures are calculated. $\lambda_{\sigma,1}, \lambda_{\sigma,2}$ are the eigenvalues of the Hessian matrix $H_\sigma(\mathbf{p})$ for a given scale parameter σ. β and c are thresholds which control the sensitivity of the line measurement. The parameter α is chosen such that the equivalent steerable filter used in the calculation of the Hessian matrix is maximally flat in its longitudinal direction.

In a similar way [2], the dominant orientation of the surface representing a curvilinear structure can be given by the dominant eigenvector of the Phase Congruency Tensor T_{PC}, i.e. the eigenvector corresponding to the eigenvalue of largest magnitude. PCT-based vesselness V_{PC} and PCT-based N_{PC} neuriteness are calculated using equations of Hessian-based vesselness and neuriteness, where the eigenvalues of T_{PC} substitute those of the Hessian.

3.2. PCT Anisotropic Diffusion

The enhancement of coherent flow-like curvilinear structures in images can be accomplished in a natural way by adopting anisotropic diffusion filtering to local texture analysis by means of the structure tensor [6]. Such a structure tensor J is defined as:

$$J(\nabla I(\mathbf{p})_\sigma)_\rho = G_\rho * \left(\nabla I(\mathbf{p})_\sigma \nabla I(\mathbf{p})_\sigma^T\right) = \begin{bmatrix} J_{11} & J_{12} \\ J_{11} & J_{22} \end{bmatrix} \quad (8)$$

where the function G_ρ denotes a Gaussian with a standard deviation ρ, and $I(\mathbf{p})_\sigma = G_\sigma * I(\mathbf{p})$ is a regularized version of $I(\mathbf{p})$ that is obtained by convolution with a Gaussian G_σ. Using eigendecomposition, $J(\mathbf{p})_\sigma$ can be expressed as follows:

$$J(\nabla I(\mathbf{p})_\sigma)_\rho = \begin{bmatrix} v_1 & v_2 \end{bmatrix} \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix} \begin{bmatrix} v_1^T \\ v_2^T \end{bmatrix} \quad (9)$$

with $\mu_1 > \mu_2$.

During an anisotropic diffusion filtering of an image, the diffusion tensor D must steer a filtering process such that the diffusion is strong mainly along the coherence direction defined by $v_1 = [v_x, v_y]$, the eigenvector corresponding to the eigenvalue of largest magnitude, and it increases with the coherence, which in [6] is defined as $(\mu_1 - \mu_2)^2$. To obtain that, D is defined as follows:

$$D = \begin{bmatrix} v_x & -v_y \\ v_y & v_x \end{bmatrix} \begin{bmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{bmatrix} \begin{bmatrix} v_x & -v_y \\ v_y & v_x \end{bmatrix}^{-1} \quad (10)$$

$$\kappa_1 = c_1$$
$$\kappa_2 = \begin{cases} c_1 & \text{if } \mu_1 = \mu_2, \\ c_1 + (1 - c_1)e^{\left(\frac{-c_2}{(\mu_1-\mu_2)^2}\right)} & \text{else} \end{cases} \quad (11)$$

where $\kappa_{1,2}$ are the eigenvalues of D, $c_1 \in (0, 1)$, $c_1 \ll 1$, and $c_2 > 0$.

We have proposed contrast independent anisotropic diffusion filtering based on the PCT concept [7], as an alternative to the structure tensor based aproach described above. To make the structure descriptor invariant under sign changes, we replace a vector field defined by PCT eigenvectors $t_{1,2}$, by its tensor product defined as follows:

$$S_{PC} = G_\rho * \begin{bmatrix} u_1^2 & u_1u_2 \\ u_1u_2 & u_2^2 \end{bmatrix} \quad (12)$$

where $u_1 = -V_{PC}t_y$ and $u_2 = V_{PC}t_x$, $[t_x, t_y]$ is the eigenvector corresponding to the eigenvalue of largest magnitude. Such defined structure tensor S_{PC} is then decomposed into its constituent eigenvectors and corresponding eigenvalues. Finally, a new PCT-based nonlinear diffusion tensor D_{PC} is constructed as described in Equation 10. Its unitary matrices are defined by the eigenvectors of S_{PC} and its diagonal matrix is given by eigenvalues defined similarly to [6]:

$$\kappa_1 = c_1$$
$$\kappa_2 = \begin{cases} c_1 & \text{if } \lambda_1 = \lambda_2, \\ c_1 + (1 - c_1)e^{\left(\frac{-c_2}{V_{PC}}\right)} & \text{else} \end{cases} \quad (13)$$

4. FEATURE EXTRACTION

4.1. PCT Live-Wire Tracing

After the application of a feature enhancement filter, a subsequent step is required to identify the curvilinear structures. This can be as simple as applying a threshold, but to ensure connectivity and improve performance in the presence of noise a number of tracing methods have been proposed.

In particular, [5] proposed a semiautomatic approach to trace 2D curvilinear structures that uses local principal ridge directions to guide the live-wire algorithm along centerlines. The cost of the path from pixel $\mathbf{p}$ to an eight-connected neighboring pixel $\mathbf{r}$ is computed using the following formula:

$$C(\mathbf{p},\mathbf{r}) = \gamma C_i(\mathbf{r}) + (1-\gamma)C_v(\mathbf{p},\mathbf{r}) \tag{14}$$

where C_i is a normalized image intensity based cost and C_v is a vector field based cost which is calculated as follows:

$$C_v(\mathbf{p},\mathbf{r}) = \frac{1}{2}\left\{\sqrt{1-\|\phi(\mathbf{p},\mathbf{r})\|} + \sqrt{1-\|\phi(\mathbf{r},\mathbf{p})\|}\right\} \tag{15}$$

$$\phi(\mathbf{p},\mathbf{r}) = |V(\mathbf{p})\cdot\mathbf{d}(\mathbf{p},\mathbf{r})| \tag{16}$$

$$\mathbf{d}(\mathbf{p},\mathbf{r}) = (\mathbf{r}-\mathbf{p})/\|\mathbf{r}-\mathbf{p}\| \tag{17}$$

where $V(\mathbf{p})$ is the value of the vector field, at the point $\mathbf{p}$. The value of $\gamma \in [0,1]$ determines the relative weight of the C_i and C_v cost components. These two components are calculated independently and the issue of scaling between them needs is addressed by normalizing both of them to the [0,1] range.

Introduction of contrast-invariant properies of the PCT approach into the live-wire tracing method has been proposed by [2]. To construct a live-wire tracing based on the PCT, the cost maps C_i for PCT vesselness and PCT neuriteness have to be calculated using Equations 2 and 3, where the eigenvalues of T_{PC} are used. In the same way, the corresponding C_v has to be calculated using the eigenvectors of T_{PC}.

4.2. PCT-Based Detection of Branching Points

A new method to detect branching points in biomedical images, without the need for image segmentation has been introduced in [8]. The proposed method relies on a vector field representation of the curve-like structures, constructed in a contrast-independent way using the PCT approach. Vector fields are then analyzed using circular statistics to identify locations where high curvature values exist along more than one principal directions. Such analysis of vector orientation is applied within a local neighborhood, defined as a circle/sphere of radius r.

Analogous to linear statistics, moments of a circular random variable are defined in terms of its probability density function [9]. Let $\Theta = \{\theta_i\}$ be a set of N angles in the range $[0, 2\pi)$. The k^{th}-order circular moment is given by:

$$m_k = \frac{\sum_{i=0}^{N-1} w_i e^{jk\theta_i}}{\sum_{i=0}^{N-1} w_i} \tag{18}$$

The orientation of m_1 represents the mean orientation; measures such as variance, skew and kurtosis can be calculated in a way analogous to scalar distributions. The weights w_i correspond to the lengths of the vectors in the distribution, which in our case we assign to traditional or PCT vesselness measures corresponding to angles θ_i.

The circular variance, which measures the spread of the probability density of the vector orientations, is closely related to the length of the mean resultant vector and is defined as follows:

$$\sigma^2 = 1 - |m_1| \tag{19}$$

Local analysis of the vector field using circular variance produces an output image which contains high values at the locations where the vector field has high curvature values, indicative of corner, junction or end points.

5. RESULTS

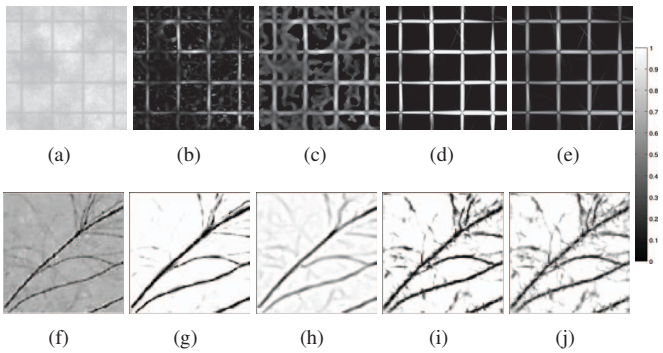


Fig. 1. Comparison between feature detection methods on synthetic image with added noise and on image of a fungal network: (a,f) original image, (b,g) vesselness, (c,h) neuriteness, (d,i) PCT vesselness, (e,j) PCT neuriteness. [2]

The performance of the PCT-based methods, discussed in the previous Sections, was evaluated on both synthetic and real images of biomedical networks.

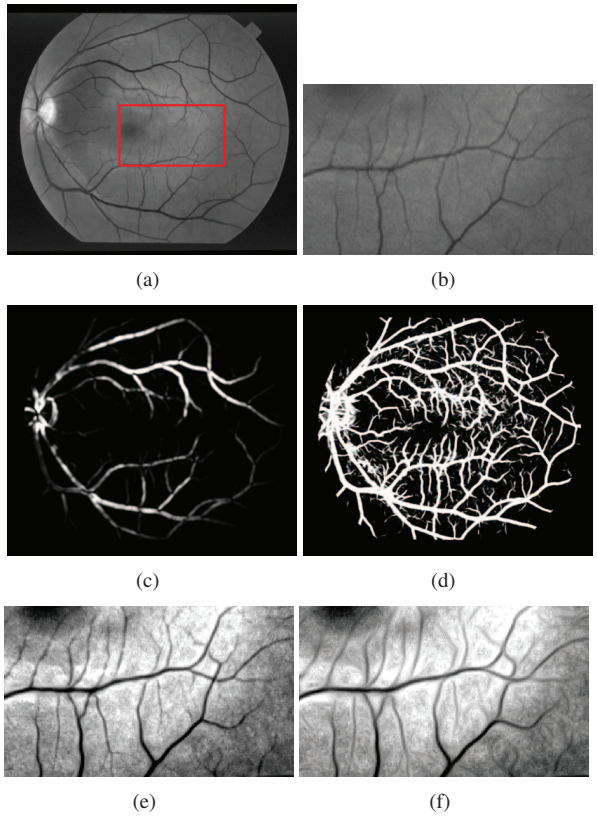
In particular, in Figure 1, we demonstrate the performance of the PCT vesselness and PCT neuriteness, for curvilinear structure enhancement on a synthetic image designed to simulate branching structures in a noisy environment, and on images of saprotrophic fungal networks. The effectiveness of using the PCT-based coherence anisotropic filtering method for retinal vascular network enhancement is presented in Figure 2.

Futhermore, the performance of the PCT-based live-wire tracing method to extract branches in images of saprotrophic fungal networks, and the PCT vector field-based method to identify branching points in retinal vascular networks is presented in Figures 3, and 4 respectively.

6. DISCUSSION AND FUTURE WORK

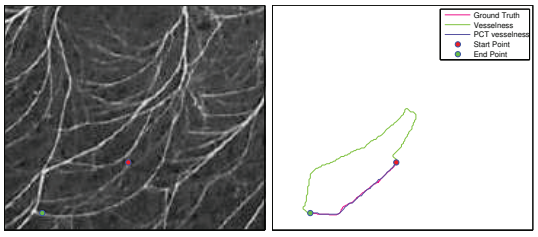
In this paper, the concept of the Phase Congruency Tensor (PCT) for curvilinear feature detection has been discussed. An immediate use of the PCT is to substitute for other tensor representations, like the Hessian, to strongly reduce the dependence on local image contrast. In particular, the PCT concept can be used to re-define curvilinear feature enhancement techniques such as: PCT vesselness [2], PCT neuriteness [2], and coherence diffusion filtering [7]. Futhermore, the PCT approach may also be used in a wide range of curvilinear and non-curvilinear feature extraction methods [2, 8].

To summarize, the obtained results show that Phase Congruency Tensor-based methods are robust against changes of intensity contrast of curvilinear structures and capable of providing high detection responses on low contrast edges (see Figures 1, 2, 3, and 4). These properties are essential for detecting the structures in low contrast regions of images, which can contain intensity inhomogeneity; these structures are common in a large number of biomedical images.



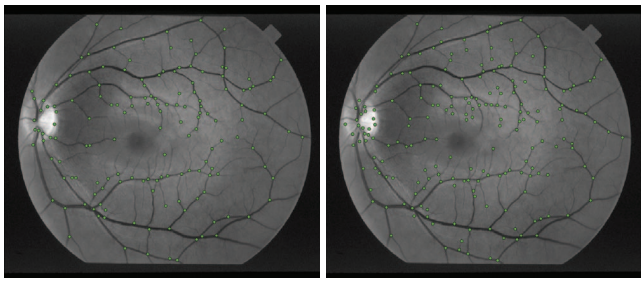
(a) (b) (c) (d) (e) (f)

Fig. 2. Comparison between feature enhancement methods on retinal vascular network images from STARE. (a) a sample image, (b) region of interest, (c,d) $(\mu_1 - \mu_2)^2$ and V_{PC} shown to illustrate variation in diffusity, (e,f) enhancement results obtained by intensity- and PCT-based approach. [7]



(a) (b)

Fig. 3. Comparison between live-wire tracing methods on fungal network images: (a) input image with the start and end points, (b) *ground truth* and obtained traces using γ=0.5 (intensity + vector field). [2]



(a) (b)

Fig. 4. Detection of feature points observed on retina images from STARE database: (a) and (b) results of the algorithm using vesselness and PCT vesselness respectively. [8]

7. REFERENCES

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